Name: 2/21/2020

Algebra 2 – Prob & Stats Unit, Permutations

Warm-up: If 7 students stand in a lunch line, in how many different ways can they line up? Factorial!

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}040 = 7!$

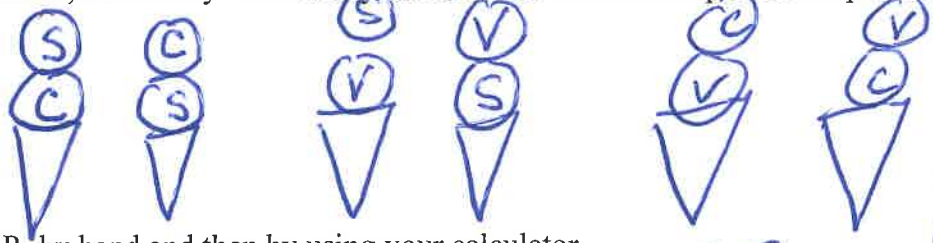
The Fundamental Counting Principle provides us with a tool that allows us to calculate the number of outcomes possible in many situations. What if the situation is a bit more complex? For many situations, the order that we complete a task does not matter. Ordering milk, bacon, and scrambled eggs in that order is the same as ordering bacon, scrambled eggs, and milk. In this case the order that we make our choices wouldn't matter, but there are many situations in which the order that we do things does make a difference.

A permutation is a specific order or arrangement of a set of objects or items. What if you wish to call someone on the phone? If I make the call, the order that I punch in the numbers matters so this is an example of a permutation. A good question to ask when deciding if your arrangement is a permutation is "DOES ORDER MATTER?" If yes, then you are dealing with a permutation. For example, if you ordered an ice cream sundae and they put the cherry in first, then the chocolate sauce, then the whip cream, and then the ice cream, you would probably not be happy with that particular ice cream sundae. You would likely prefer that they put the ice cream in first, then the chocolate sauce, then the whip cream, and then put the cherry on top. Clearly each sundae had the same four ingredients, but they were quite different from one another. Each order that we can make the ice cream sundae is called a permutation.

Example 1

Suppose you are going to order an ice cream cone with two different flavored scoops. You are going to take a picture of your ice cream cone for use in the school newspaper. The ice cream shop has 3 flavors to choose from; chocolate, vanilla, strawberry. How many different ice cream cone photos are possible?

Solution



$_3P_2 = \frac{3!}{(3!-2!)} = \frac{3!}{1!}$

Example 2

Give the value of $_6P_4$ by hand and then by using your calculator.

$_6P_4 = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360 \quad = \frac{3 \times 2 \times 1}{1} = 6$

Example 3

In a class of 32 students, in how many ways could you choose a valedictorian, president, and secretary?

$_{32}P_3 = 32 \cdot 31 \cdot 30$ or $\frac{32!}{29!} = 29{,}760$

Example 4

If 12 cars compete in a race, in how many ways can any four cars finish 1st, 2nd, 3rd, and 4th?

$_{12}P_4$ or $12 \cdot 11 \cdot 10 \cdot 9$ or $\frac{12!}{8!}$

Example 5

How many possible anagrams are there using the word "math"?

$_4P_4 \rightarrow 4 \times 3 \times 2 \times 1$ or $\frac{4!}{0!} = 4!$