

Key

3/6/2020

2.5 Conditional Probabilities and 2-Way Tables

Learning Objectives

- Understand how to calculate conditional probabilities
- Understand how to calculate probabilities using a contingency or 2-way table

It is quite easy to calculate simple probabilities. What is the chance of rolling a 4 with a single die? What is the chance of being dealt a queen from a deck of cards? We are now going to focus on conditional probabilities. A **conditional probability** is a probability in which a certain prerequisite condition has already been met.

We can start by thinking about cards being dealt from a standard deck of 52 cards. As each card is dealt, what remains in the deck changes. A gambler in a casino will pay close attention to cards played. If many face cards have already been dealt, the observant gambler will understand that the next card has a higher chance of not being a face card. Suppose we want to know the probability that our next card will be a face card given that the first card was the 7 of diamonds. The formal notation for this is $P(\text{Face}|\text{7Diamond})$. This is read as "The probability of a face card given that we already have been dealt the 7 of diamonds.". Often times the math for these situations is very logical. In our case, we have simply reduced the deck by one card and there are still 12 face cards in the deck. Therefore $P(\text{Face}|\text{7Diamond}) = 12/51 = 4/17 = 0.24$.

Example 1

Two cards are dealt from a standard deck of 52 cards. Find each conditional probability.

a) $P(\text{2nd red}|\text{1st 2Clubs})$

b) $P(\text{2nd red}|\text{1st 2Diamonds})$

c) $P(\text{2nd club}|\text{1st red})$

Solution

Take out 2 of Clubs (black)
So, $P(\text{2nd card red}) = \frac{26}{51}$

Remove
 $P(\text{2nd card red}) = \frac{25}{51}$

Remove a red
 $P(\text{2nd card club}) = \frac{13}{51}$

Example 2

In a common poker game, 5 cards are dealt to a player. The best possible hand is called a royal flush. This occurs if a player gets the ten, jack, queen, king, and ace all of the same suit. What is the chance of being dealt a royal flush? Leave your answer as a fraction.

Solution

any of these - there are four of each, could "kick off" the royal flush!

$$\frac{20}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48} = \frac{480}{311875200}$$

OR $\approx .00015\%$
Wow!!

Now you're locked into a suit, 4 "winners" remain

Example 3

Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Suppose we randomly select one student.

	Bus	Walk	Car	Other	TOTALS
9 th or 10 th grade	106	30	70	4	210
11 th or 12 th grade	41	58	184	7	290
TOTALS	147	88	254	11	500

a) Give all the row and column totals.

b) What is the probability that the student walked to school? $\frac{88}{500}$

c) What is the probability that the student was a 9th or 10th grader? $\frac{210}{500}$

d) What is the probability that a student either rode the bus or is in 11th or 12th grade?

see it? $\frac{290 + 147 - 41 \text{ overlap}}{500} \rightarrow \frac{396}{500}$

Grand TOTAL

Example 4

Consider the completed chart in the solution of part a) of Example 3.

a) What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

$$\frac{184}{254}$$

b) What is $P(\text{Walk} | 9\text{th or } 10\text{th grade})$? $\frac{30}{210}$

Solution

The "given" is your Denominator

Example 5

The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a contingency table that tracks the type of customer and type of ice cream.

	Chocolate	Vanilla	TOTALS
adults	41	52	93
children	105	26	131
TOTALS	146	78	224

Example 6

A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

Solution

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22
45

