

Name Hansen Hour 4 Date 1/29/2020

Algebra 2: Equation of a CIRCLE, via "completing the square"

Equation of a Circle: $(x - h)^2 + (y - k)^2 = r^2$

Where (h, k) is the center and r is the radius.

Warm-up

Use the information provided to write the standard form equation of each circle.

- 1) Center: $(-5, 2)$
Radius: 8

$$(x + 5)^2 + (y - 2)^2 = 64$$

- 2) Center: $(12, 10)$
Radius: 2

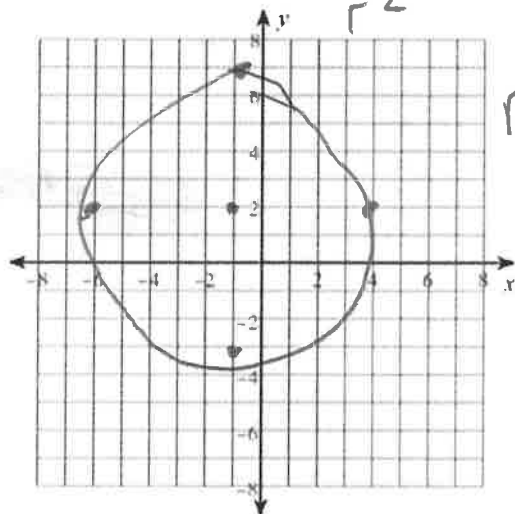
$$(x - 12)^2 + (y - 10)^2 = 4$$

Identify the center and radius of each. Then sketch the graph.

3) $(x + 1)^2 + (y - 2)^2 = 25$

C: $(-1, 2)$

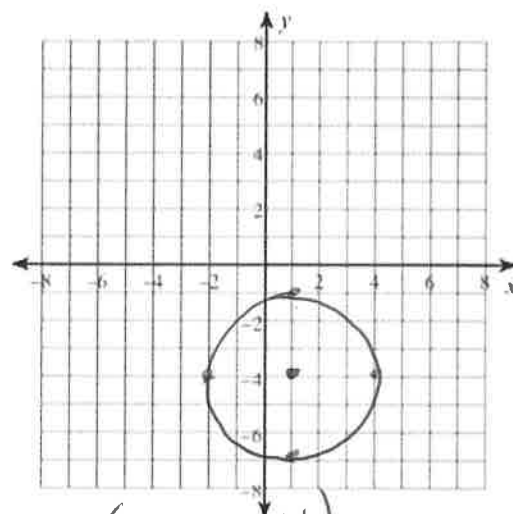
$r = \sqrt{25} = 5$



4) $(x - 1)^2 + (y + 4)^2 = 9$

C: $(1, -4)$

$r = 3$



So: We know how to identify the center and a radius of a circle when given the following format:

CIRCLE: $(x - 3)^2 + (y + 2)^2 = 25$

C: $(\underline{3}, \underline{-2})$ $r = \underline{5}$ ✓

However, sometimes the equation isn't in this standard form!

For Example: $x^2 + y^2 - 4x + 6y - 12 = 0$ IS the equation of a circle! BUT, it *seems* impossible to tell what's the center (h, k) and the radius, r, in this particular format. Take a guess.... I dare ya!

So, we have to manipulate this equation to make it look like the standard: $(x - h)^2 + (y - k)^2 = r^2$

This process of making the equation look like standard form is called "Completing the Square."

Example One: $x^2 + y^2 - 4x + 6y - 12 = 0$

① Re-organize $(x^2 - 4x + \boxed{4}) + (y^2 + 6y + \boxed{9}) = 12 + \boxed{4} + \boxed{9}$

② Get # in Box
Take half coefficient,
Then square it.

③ make a binomial square

Half, square it

Cut in Half

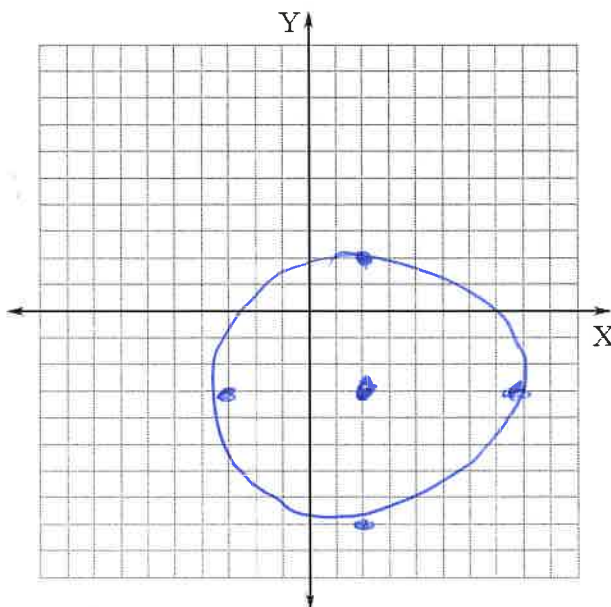
Cut Half

TOTAL 25

$(x - 2)^2 + (y + 3)^2 = 25$

C: (2, -3)

r: $\sqrt{25} = 5$



Example Two: $x^2 + y^2 + 8x + 2y + 8 = 0$

$$(x^2 + 8x + \boxed{16}) + (y^2 + 2y + \triangle 1) = \overset{\#}{-8} + \boxed{16} + \triangle 1$$

$$\downarrow \qquad \qquad \downarrow$$

$$(x+4)^2 + (y+1)^2 = 9$$

Example Three: $x^2 + y^2 + 6x - 8y + 24 = 0$

$$(x^2 + 6x + \boxed{9}) + (y^2 - 8y + \triangle 16) = -24 + \boxed{9} + \triangle 16$$

$$\downarrow$$

$$(x+3)^2 + (y-4)^2 = 1$$

Example Four: $x^2 + y^2 + 2y - 24 = 0$

$$x^2 + (y^2 + 2y + \triangle 1) = 24 + \triangle 1$$

$$\downarrow \qquad \qquad \downarrow$$

$$x^2 + (y+1)^2 = 25$$

