2.5 Conditional Probabilities and 2-Way Tables

Learning Objectives

• Understand how to calculate conditional probabilities • Understand how to calculate probabilities using a contingency or 2-way table

It is quite easy to calculate simple probabilities. What is the chance of rolling a 4 with a single die? What is the chance of being dealt a queen from a deck of cards? We are now going to focus on conditional probabilities. A **conditional probability** is a probability in which a certain prerequisite condition has already been met.

We can start by thinking about cards being dealt from a standard deck of 52 cards. As each card is dealt, what remains in the deck changes. A gambler in a casino will pay close attention to cards played. If many face cards have already been dealt, the observant gambler will understand that the next card has a higher chance of not being a face card. Suppose we want to know the probability that our next card will be a face card given that the first card was the 7 of diamonds. The formal notation for this is P(Face|7Diamond). This is read as "The probability of a face card given that we already have been dealt the 7 of diamonds.". Often times the math for these situations is very logical. In our case, we have simply reduced the deck by one card and there are still 12 face cards in the deck. Therefore P(Face|7Diamond)= 12/51 = 4/17 = 0.24.

Example 1

Two cards are dealt from a standard deck of 52 cards. Find each conditional probability.

a) P(2nd red|1st 2Clubs) b) P(2nd red|1st 2Diamonds) c) P(2nd club|1st red)

Solution

Example 2

In a common poker game, 5 cards are dealt to a player. The best possible hand is called a royal flush. This occurs if a player gets the ten, jack, queen, king, and ace all of the same suit. What is the chance of being dealt a royal flush? Leave your answer as a fraction.

Solution

Example 3

Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Suppose we randomly select one student.

	Bus	Walk	Car	Other
9^{th} or 10^{th} grade	106	30	70	4
11^{th} or 12^{th} grade	41	58	184	7

a) Give all the row and column totals.

b) What is the probability that the student walked to school?

c) What is the probability that the student was a 9th or 10th grader?

d) What is the probability that a student either rode the bus or is in 11th or 12th grade?

Example 4

Consider the completed chart in the solution of part a) of Example 3.

a) What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

b) What is P(Walk|9th or 10th grade)?

Solution

Example 5

The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a contingency table that tracks the type of customer and type of ice cream.

Solution

Example 6

A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the probability that a randomly selected student listens to pop music given that they listen country music.

Solution

Name: _____

Problem Set 2.5

Exercises

1) Figure 2.2 shows the counts of earned degrees for several colleges on the East Coast. The level of degree and the gender of the degree recipient were tracked. Row & Column totals are included.

	Bachelor's	Master's	Professional	Doctorate	Total
Female	542	128	26	18	**714**
Male	438	165	38	20	**661**
Total	**980**	**293**	**64**	**38**	**1375**

FIGURE 2.3

a) What is the probability that a randomly selected degree recipient is a female?

b) What is the probability that a randomly chosen degree recipient is a male?

c) What is the probability that a randomly selected degree recipient is a woman, given that they received a Master's Degree?

d) For a randomly selected degree recipient, what is P(Bachelor's Degree|Male)?

2) In poker, 5 cards are dealt to a player. One of the stronger poker hands is a flush. This means that all 5 cards are of the same suit, for example, all hearts. What is the probability of being dealt a flush?

3) The table below shows the probability breakdown of ages and genders for the typical American college student. Each value in the table is given as a probability. For example, there is a 12% chance that a randomly selected college student will be a male between 25 and 34 years old.

TABLE 2.3:

	14-17	18-24	25-34	>34
Male	.01	.30	.12	.04
Female	.01	.30	.13	.09

a) What is the probability that a randomly selected American college student is female?

b) What is the probability that a randomly selected American college student is female given that the student is more than 34 years old?

c) What is the probability that a randomly selected college student is either a female or more than 34 years old?

4) Suppose that 40% of adults like eating bananas while 60% like eating apples. Suppose also that 32% of adults like eating both. What is the conditional probability that a randomly selected adult likes apples given that they like bananas? Use a Venn Diagram to answer this question.

5) Another good poker hand is called a straight. This means that your five cards will be numerically in order such as an 8, 9, 10, jack, and queen. The cards do not need to match suit in a straight. Suppose you receive the first four cards of a five card poker hand. You have 5Heart, 7Diamond, 8Club, and 9Diamond. What is the probability that the next card will give you a straight?

6) Suppose you receive the first four cards of a five card poker hand. You have 3Heart, 4Diamond, 5Club, and 6Diamond. What is the probability that your next card will give you a straight?

7) A statistics class has 18 juniors and 10 seniors in it. 6 of the seniors are females and 12 of the juniors are males. Build a contingency table to find the probability that a randomly selected student is:

a) a junior or a female?
b) a senior or a female?
c) a junior or a senior?
d) a female given that the student was a senior?

8) At a used-book sale, there are 120 children's books and 80 adult books available. 50 of the adult books are nonfiction while 40 of the children's books are nonfiction. All other books are fiction. Build a contingency table to find the probability that a randomly selected book is:
a) fiction.
b) not a children's nonfiction
c) an adult book or children's nonfiction.

9) Animals on the endangered species list are given in the table below by type of animal and whether it is domestic or foreign to the United States.

TABLE 2.4:

	Mammals	Birds	Reptiles	Amphibians
United States	63	78	14	10
Foreign	251	175	64	8

An endangered animal is selected at random. What is the probability that it is:

a) a bird found in the United States?

b) foreign or a mammal?

c) a bird given that it is found in the United States?

d) a bird given that it is foreign?

10) Suppose a standard set of pool balls (1-8 are solid and 9-15 are striped) are in a bag. Two pool balls are picked out of the bag without replacement.

a) Find the probability that second ball is striped given that the first ball was the 10 ball.

b) Find P(2nd striped|1st solid).

11) Cable channels 6, 8, & 10 show quiz shows, comedies, & dramas. The table below shows the distributions of these shows.

TABLE 2.5:

	Channel 6	Channel 8	Channel 10
Quiz Show	4	2	1
Comedy	3	3	8
Drama	4	5	1

If a show is selected at random, find the probability that the show is:

a) a quiz show or shown on Channel 8.

b) a drama or a comedy.

c) a comedy given that it is shown on Channel 8.

d) shown on Channel 6 given that it is a drama.

12) Suppose you receive your first three cards of a five card poker hand.

You have 5Diamond, 6Diamond, 7Heart. What is the probability that your next two cards will result with you having a straight?