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KEY **Precalculus**

Chapter 7 Study Guide

HansenMath Precalculus students:

As you prepare for the Chapter 7 Unit Test on Thursday, here are a few things to be aware of:

- Any of the concepts/problems assigned in class from sections 7.2 → 7.8 are fair game.
- However, the attached review should serve you well as a study guide.
- I am going to allow you to have your graphing calculator out for the duration of this test. However, there are some stipulations:
  - If the problem says "solve by hand," you MUST show all work, or forfeit all points for that problem.
  - If the problem says "by calc" - or doesn't say anything - you're safe to assume you can use your calculator, but you MUST write "used calc" next to your setup work and/or answer.
  We'll practice this on this assignment ☺
- Let's do this thing!

(Note: If there's a problem number given, it's from the Chapter 7 Review in our textbook)

Section 7.2 Systems of Equations

Solve the system, by hand.

27. $\begin{cases} 1.25x - 2y = 3.5 \\ 5x - 8y = 14 \end{cases}$

(-4)EQ1: $-5x + 8y = -14$
EQ2: $5x - 8y = 14$
$0 = 0$

$*$ infinitely many solutions

17. ***Break-Even Analysis*** You set up a business and make an initial investment of \$10,000. The unit cost of the product is \$2.85 and the selling price is \$4.95. How many units must you sell to break even?

Revenue (Money in) = $4.95x$
Cost (money out) = $2.85x + 10{,}000$

Break even when cost = revenue
$2.85x + 10{,}000 = 4.95x$
$x \approx 4{,}762$ units

Section 7.3 Multivariable Linear Systems

By hand, use back substitution to solve.

39. $\begin{cases} x - 4y + 3z = -14 \\ \quad\; - y + z = -5 \\ \qquad\quad\;\; z = -2 \end{cases}$

$-y - 2 = -5$

$-y = -3$

$y = 3$

$x - 4(3) + 3(-2) = -14$

$x - 12 - 6 = -14$

$x - 18 = -14$

$x = 4$

Solution: $(4, 3, -2)$

By hand, use elimination & ~~back~~ substitution to solve.

45. $\begin{cases} x - 2y + 3z = -5 \\ 2x + 4y + 5z = 1 \\ x + 2y + z = 0 \end{cases}$

(-2)EQ1: $-2x + 4y - 6z = 10$

EQ2: $2x + 4y + 5z = 1$

$\boxed{8y - z = 11}$

(-1)EQ1: $-x + 2y - 3z = 5$

EQ3: $x + 2y + z = 0$

$\boxed{4y - 2z = 5}$

(-2): $-8y + 4z = -10$

$8y - z = 11$

$3z = 1 \rightarrow \boxed{z = \frac{1}{3}} \rightarrow$ sub $\rightarrow \boxed{y = \frac{17}{12}} \rightarrow \boxed{x = \frac{-19}{6}}$

59. ***Agriculture*** A mixture of 6 gallons of chemical A, 8 gallons of chemical B, and 13 gallons of chemical C is required to kill a destructive crop insect. Commercial spray X contains 1, 2, and 2 parts, respectively, of these chemicals. Commercial spray Y contains only chemical C. Commercial spray Z contains chemicals A, B, and C in equal amounts. How much of each type of commercial spray is needed to obtain the desired mixture?

Tough!

Chemical A: $\frac{1}{5}x + 0y + \frac{1}{3}z = 6$

Chemical B: $\frac{2}{5}x + 0y + \frac{1}{3}z = 8$

Chemical C: $\frac{2}{5}x + 1y + \frac{1}{3}z = 13$

using [A] → RREF[A]

$x = 10$ gal, $y = 5$ gal, $z = 12$ gal

Section 7.4 Matrices and Gaussian Elimination

7.4 In Exercises 61–64, determine the order of the matrix.

61. $\begin{bmatrix} -3 \\ 1 \\ 10 \end{bmatrix}$ 3×1

62. $\begin{bmatrix} 3 & -1 & 0 & 6 \\ -2 & 7 & 1 & 4 \end{bmatrix}$ 2×4

63. $[14]$ 1×1

64. $[6 \quad 7 \quad -5 \quad 0 \quad -8]$ 1×5

Write as an "augmented Matrix"

67. $\begin{cases} 8x - 7y + 4z = 12 \\ 3x - 5y + 2z = 20 \\ 5x + 3y - 3z = 26 \end{cases}$

↓

$$\left[\begin{array}{ccc|c} 8 & -7 & 4 & 12 \\ 3 & -5 & 2 & 20 \\ 5 & 3 & -3 & 26 \end{array}\right]$$

By hand, re-write in "row echelon" form

72. $\begin{bmatrix} 3 & 5 & 2 \\ 1 & -2 & 4 \\ -2 & 0 & 5 \end{bmatrix}$

Want Look $\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{array}{l} R1 \\ R2 \\ R3 \end{array}\begin{bmatrix} 1 & -2 & 4 \\ 3 & 5 & 2 \\ -2 & 0 & 5 \end{bmatrix} \rightarrow \begin{array}{r} R1: \\ -3R1+R2 \\ +2R1+R3 \end{array}\begin{bmatrix} 1 & -2 & 4 \\ 0 & 11 & -10 \\ 0 & -4 & 13 \end{bmatrix}$

$\begin{array}{l} R1 \\ R2 \\ R2+\frac{11}{4}R3 \end{array}\begin{bmatrix} 1 & -2 & 4 \\ 0 & 11 & -10 \\ 0 & 0 & \frac{103}{4} \end{bmatrix} \rightarrow \begin{array}{r} R1: \\ (\frac{1}{11})R2: \\ (\frac{4}{103})R3: \end{array}\begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -\frac{10}{11} \\ 0 & 0 & 1 \end{bmatrix}$

Use calc to re-write matrix in RREF form

75. $\begin{bmatrix} 1.5 & 3.6 & 4.2 \\ 0.2 & 1.4 & 1.8 \\ 2.0 & 4.4 & 6.4 \end{bmatrix} \rightarrow [A]$

RREF [A]

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By hand, write as matrix and use Gaussian Elimination to solve.

82. $\begin{cases} x + 2y + 6z = 1 \\ 2x + 5y + 15z = 4 \\ 3x + y + 3z = -6 \end{cases}$

$\begin{bmatrix} 1 & 2 & 6 & 1 \\ 2 & 5 & 15 & 4 \\ 3 & 1 & 3 & -6 \end{bmatrix} \rightarrow \begin{array}{r} R1: \\ (-2)R1+R2: \\ (-3)R1+R3: \end{array}\left[\begin{array}{ccc:c} 1 & 2 & 6 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -5 & -15 & -9 \end{array}\right]$

$\rightarrow \begin{array}{r} R1: \\ R2: \\ 5R2+R3: \end{array}\left[\begin{array}{ccc:c} 1 & 2 & 6 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right] \rightarrow 0 \neq 1$

No solution

Use calc and matrix RREF to solve the system.

86. $\begin{cases} 4x + 4y + 4z = 5 \\ 4x - 2y - 8z = 1 \\ 5x + 3y + 8z = 6 \end{cases}$

$\begin{bmatrix} 4 & 4 & 4 & 5 \\ 4 & -2 & -8 & 1 \\ 5 & 3 & 8 & 6 \end{bmatrix} \rightarrow [A]$

RREF [A]

$\left(\frac{31}{42}, \frac{5}{14}, \frac{13}{84}\right)$

Section 7.5 Matrix Operations

Find the values for x & y

94. $\begin{bmatrix} -1 & 0 \\ x & 5 \\ -4 & y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 8 & 5 \\ -4 & 0 \end{bmatrix}$

$x = 8$

$y = 0$

Find C = 5A - 3B and then find C_{23}

$A = \begin{bmatrix} 6 & 0 & 7 \\ 5 & -1 & 2 \\ 3 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 & 1 \\ -4 & 8 & 6 \\ 2 & -1 & 1 \end{bmatrix}$

$5A = \begin{bmatrix} 30 & 0 & 35 \\ 25 & -5 & 10 \\ 15 & 10 & 15 \end{bmatrix} \quad -3B = \begin{bmatrix} 0 & -15 & -3 \\ 12 & -24 & -18 \\ -6 & 3 & -3 \end{bmatrix}$

$5A - 3B: \begin{bmatrix} 30 & -15 & 32 \\ 37 & -29 & -8 \\ 9 & 13 & 12 \end{bmatrix} \quad C_{23} = -8$

By hand, multiply

In Exercises 111–114, find AB, if possible.

111. $A = \begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix}$

3x2   2 x 3

112. $A = \begin{bmatrix} 1 & 5 & 6 \\ 2 & -4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 5 & 2 \\ 0 & 1 & 0 \end{bmatrix}$

2x3   2x3

111. $AB = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} 6+8 & -2+0 & 8+0 \\ 30-16 & -10+0 & 40+0 \\ 36+0 & -12+0 & 48+0 \end{bmatrix} \end{array} \rightarrow \begin{bmatrix} 14 & -2 & 8 \\ 14 & -10 & 40 \\ 36 & -12 & 48 \end{bmatrix}$

112. undefined

By calc, multiply

[A] ([B] * [C])

118. $\begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & -3 \end{bmatrix}\right)$

$\rightarrow \begin{bmatrix} 4 & -3 \\ 82 & -48 \end{bmatrix}$

120. ***Exercise*** The numbers of calories burned by individuals of different weights performing different types of aerobic exercises for 20-minute time periods are shown in the matrix.

$$B = \begin{array}{c} \begin{array}{cc} \text{120-lb person} & \text{150-lb person} \end{array} \\ \begin{bmatrix} 109 & 136 \\ 127 & 159 \\ 64 & 79 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Bicycling} \\ \text{Jogging} \\ \text{Walking} \end{array}$$

(a) A 120-pound person and a 150-pound person bicycle for 40 minutes, jog for 10 minutes, and walk for 60 minutes. Organize a matrix A for the time spent exercising in units of 20-minute intervals.

(b) Find the product AB.

(c) Explain the meaning of the product AB in the context of the situation.

# of 20 min intervals

$A = \begin{array}{c} \begin{array}{ccc} \text{Bike} & \text{Jog} & \text{Walk} \end{array} \\ \begin{bmatrix} 2 & \frac{1}{2} & 3 \end{bmatrix} \end{array}$ 1 x3

$B = \begin{array}{c} \begin{array}{cc} \text{120lb} & \text{150Lb} \end{array} \\ \begin{bmatrix} 109 & 136 \\ 127 & 159 \\ 64 & 79 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Bike} \\ \text{Jog} \\ \text{walk} \end{array}$ 3x2

yields 1x2

TOTAL Cal burned $\begin{array}{c} \begin{array}{cc} \text{120lb} & \text{150lb} \end{array} \\ \begin{bmatrix} 473.5 & 588.5 \end{bmatrix} \end{array}$

True or False Questions

Matrix Multiplication is commutative?

False – Not always

Matrices must have the same order to multiply?

False

Matrix addition is commutative?

True

Matrices must have the same order to add? True

All Matrices have an Inverse?

False – only some square Matrices

All Matrices have a Determinant?

False – only square matrices

Section 7.6 Inverse of a Square Matrix

By hand, show that Matrix B is the inverse of A

121. $A = \begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix}$

$A * B$ must $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A * B = \begin{bmatrix} 8-7 & 4-4 \\ -14+14 & -7+8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity

$\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

By hand, find the inverse matrix, if it exists.

123. $\begin{array}{cc} a & b \\ \begin{bmatrix} -6 & 5 \\ -5 & 4 \end{bmatrix} \\ c & d \end{array}$

$\dfrac{1}{-24--25}\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

$1\begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

124. $\begin{array}{cc} a & b \\ \begin{bmatrix} -3 & -5 \\ 2 & 3 \end{bmatrix} \\ c & d \end{array}$

$\dfrac{1}{-9--10}\begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

By calc, find the inverse matrix, if it exists.

129. $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow [A]$

$[A]^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & 1 & 3 \end{bmatrix}$

Use any method by calculator to solve the system

139. $\begin{cases} x + 2y + z - w = -2 \\ 2x + y + z + w = 1 \\ x - y - 3z = 0 \\ z + w = 1 \end{cases}$

Matrix $[A] \rightarrow$ RREF $[A]$

$= (1, -2, 1, 0)$

Section 7.7 Determinant of a Square Matrix

$ad - bc$

By hand, find the determinant

145. $\begin{array}{cc} a & b \\ \begin{bmatrix} 8 & 5 \\ 2 & -4 \end{bmatrix} \\ c & d \end{array} \rightarrow -32-10 = -42$ det

147. $\begin{array}{cc} a & b \\ \begin{bmatrix} 50 & -30 \\ 10 & 5 \end{bmatrix} \\ c & d \end{array} \rightarrow 250--300 = 550$ det

Find all ~~majors~~ minors and ~~minors~~ Cofactors of the matrix

151. $\begin{bmatrix} 3 & 2 & -1 \\ -2 & 5 & 0 \\ 1 & 8 & 6 \end{bmatrix}$

$M_{11} = 30 - 0 = 30$ $\quad M_{12} = -12 - 0 = -12$ $\quad M_{13} = -16 - 5 = -21$

$M_{21} = 12 - -8 = 20$ $\quad M_{22} = 18 - -1 = 19$ $\quad M_{23} = 24 - 2 = 22$

$M_{31} = 0 - -5 = 5$ $\quad M_{32} = 0 - 2 = -2$ $\quad M_{33} = 15 - -4 = 19$

$C_{11} = 30$ $\quad C_{12} = 12$ $\quad C_{13} = -21$

$C_{21} = 20$ $\quad C_{22} = 19$ $\quad C_{23} = 22$ Cofactors

$C_{31} = 5$ $\quad C_{32} = 2$ $\quad C_{33} = 19$ * apply sign changes

By hand, find the determinant.
Expand by cofactors on the row of your choice.

153. $\begin{bmatrix} -2 & 4 & 1 \\ -6 & 0 & 2 \\ 5 & 3 & 4 \end{bmatrix} \leftarrow$ I Chose Row 2

Find Cofactors

sign −
$C_{21} = 16 - 3 = -13$

*sign +
$C_{22} = -16 - 5 = -21$

sign −
$C_{23} = -6 - 20 = 26$

$\det = a_{21} \cdot C_{21} + a_{22} \cdot C_{22} + a_{23} \cdot C_{23}$
$= -6 \cdot -13 + 0 \cdot -21 + 2 \cdot 26$
$= 78 + 0 + 52$
$= 130$

Find the determinant - use your fav method

159. $\begin{vmatrix} 8 & 6 & 0 & 2 \\ 0 & -1 & 1 & -4 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 3 \end{vmatrix}$

use calc det

OR

upper Triangle
$(8)(-1)(4)(3)$
$\det = -96$

Section 7.8 Applications of the Determinant, and Cryptography

Use the determinant to find the area given by:

165. (2, 4), (5, 6), (4, 1)

Area $= \frac{1}{2} \cdot \det \begin{vmatrix} 2 & 4 & 1 \\ 5 & 6 & 1 \\ 4 & 1 & 1 \end{vmatrix}$

$= -\frac{1}{2} \cdot -13 = \frac{13}{2}$ or $6.5u^2$

Use the determinant to find the area given by:

167. (−2, −1), (4, 9), (−2, −9), (4, 1)

Area $= \det \begin{vmatrix} -2 & -1 & 1 \\ 4 & 9 & 1 \\ -2 & -9 & 1 \end{vmatrix}$

$= 48\ u^2$

In Exercises 169 and 170, use a determinant to determine whether the points are collinear.

169. (−1, 7), (2, 5), (4, 1)

$\det \begin{vmatrix} -1 & 7 & 1 \\ 2 & 5 & 1 \\ 4 & 1 & 1 \end{vmatrix}$

$\det = -8$

Not collinear

170. (0, −5), (2, 1), (4, 7)

$\det \begin{vmatrix} 0 & -5 & 1 \\ 2 & 1 & 1 \\ 4 & 7 & 1 \end{vmatrix}$

$\det = 0$

Collinear!!

In Exercises 181 and 182, write the uncoded 1×3 row matrices for the message. Then encode the message using the encoding matrix.

Code w/ alphabet # chart

Message	*Encoding Matrix*
181. I HAVE A DREAM	$\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} \rightarrow [B]$
182. JUST DO IT	$\begin{bmatrix} 2 & 1 & 0 \\ -6 & -6 & -2 \\ 3 & 2 & 1 \end{bmatrix} \rightarrow [B]$

In Exercises 183–186, use A^{-1} to decode the cryptogram.

183. $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & -2 & 0 \\ 1 & -2 & 2 \end{bmatrix}$ 32 −46 37 9 −48 15 3 −14 10 −1 −6 2 −8 −22 −3

184. $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 2 & -1 & 2 \end{bmatrix}$ 30 −7 30 5 10 80 37 34 16 40 −7 38 −3 8 36 16 −1 58 23 46 0

181 5x3

$$\begin{bmatrix} I & \_ & H \\ A & V & E \\ \_ & A & \_ \\ D & R & E \\ A & M & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 9 & 0 & 8 \\ 1 & 22 & 5 \\ 0 & 1 & 0 \\ 4 & 18 & 5 \\ 1 & 13 & 0 \end{bmatrix} \xrightarrow{\text{Set as}} [A] * [B] \overset{\text{encoding}}{=} \overset{\text{Crypto}}{\begin{bmatrix} -30 & -2 & 24 \\ 38 & 8 & -51 \\ 3 & 0 & -3 \\ 32 & 2 & -39 \\ 41 & -2 & -39 \end{bmatrix}}$$

182.

$$\begin{bmatrix} J & U & S \\ T & \_ & D \\ O & \_ & I \\ T & \_ & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 21 & 19 \\ 20 & 0 & 4 \\ 15 & 0 & 9 \\ 20 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Set as}} [A] * [B] \overset{\text{encoding}}{=} \overset{\text{Cryptogram}}{\begin{bmatrix} -49 & -78 & -23 \\ 52 & 28 & 4 \\ 57 & 33 & 9 \\ 40 & 20 & 0 \end{bmatrix}}$$

(183.) let $[B] = \overset{5\times 3}{\begin{bmatrix} 32 & -46 & 37 \\ 9 & -48 & 15 \\ 3 & -14 & 10 \\ -1 & -6 & 2 \\ -8 & -22 & -3 \end{bmatrix}} * [A]^{-1} \rightarrow$

$\rightarrow \begin{bmatrix} 9 & 0 & 23 \\ 9 & 12 & 12 \\ 0 & 2 & 5 \\ 0 & 2 & 1 \\ 3 & 11 & 0 \end{bmatrix} = \begin{matrix} I & \_ & W \\ I & L & L \\ \_ & B & E \\ \_ & B & A \\ C & K & \_ \end{matrix} \rightarrow$ "I will be back"

(184.) let $[B] = \overset{7\times 3}{\begin{bmatrix} 30 & -7 & 30 \\ 5 & 10 & 80 \\ 37 & 34 & 16 \\ 40 & -7 & 38 \\ -3 & 8 & 36 \\ 16 & -1 & 58 \\ 23 & 46 & 0 \end{bmatrix}} * [A]^{-1} \rightarrow$

$\rightarrow \begin{bmatrix} 3 & 1 & 14 \\ 0 & 25 & 15 \\ 21 & 0 & 8 \\ 5 & 1 & 18 \\ 0 & 13 & 5 \\ 0 & 14 & 15 \\ 23 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} C & A & N \\ \_ & Y & O \\ U & \_ & H \\ E & A & R \\ \_ & M & E \\ \_ & N & O \\ W & \_ & \_ \end{bmatrix} \rightarrow$ "Can You Hear me Now"