

Algebra 2

Section 7-5

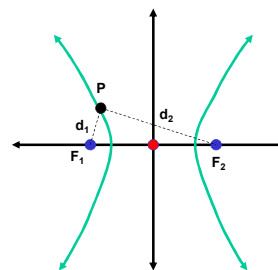
Hyperbolas

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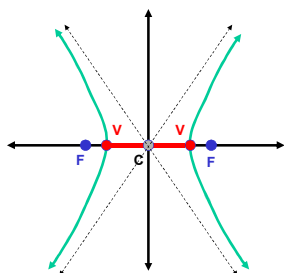
A **hyperbola** is a set of points in a plane the difference of whose distances from two fixed points, called **foci**, is a constant.

★ For any point P that is on the hyperbola, $d_2 - d_1$ is always the same.

★ In this example, the origin is the **center** of the hyperbola. It is midway between the foci.



Hyperbolas



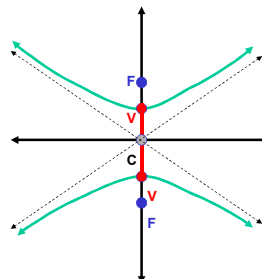
★ A line through the foci intersects the hyperbola at two points, called the **vertices**.

★ The segment connecting the vertices is called the **transverse axis** of the hyperbola.

★ The **center** of the hyperbola is located at the midpoint of the transverse axis.

★ As x and y get larger the branches of the hyperbola approach a pair of intersecting lines called the **asymptotes** of the hyperbola. These asymptotes pass through the center of the hyperbola.

Hyperbolas



The figure at the left is an example of a hyperbola whose branches open up and down instead of right and left.

★ Since the transverse axis is vertical, this type of hyperbola is often referred to as a **vertical hyperbola**.

★ When the transverse axis is horizontal, the hyperbola is referred to as a **horizontal hyperbola**.

Standard Form Equation of a Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Horizontal
Hyperbola

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Vertical
Hyperbola

★ The center of a hyperbola is at the point (h, k) in either form

★ For either hyperbola, $c^2 = a^2 + b^2$

Where c is the distance from the center to a focus point.

★ The equations of the asymptotes are

$$y = \pm \frac{b}{a}(x-h) + k \quad \text{and} \quad y = \pm \frac{a}{b}(x-h) + k$$

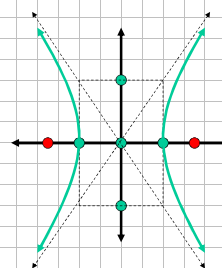
for horizontal hyperbola for vertical hyperbola

Graphing a Hyperbola

Graph: $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Center: (0, 0)

- ★ The x-term comes first in the subtraction so this is a horizontal hyperbola
- ★ From the center locate the points that are two spaces to the right and two spaces to the left
- ★ From the center locate the points that are up three spaces and down three spaces
- ★ Draw a dotted rectangle through the four points you have found.
- ★ Draw the asymptotes as dotted lines that pass diagonally through the rectangle.
- ★ Draw the hyperbola.



$$c^2 = 9 + 4 = 13$$

$$c = \sqrt{13} = 3.61$$

Vertices: (2, 0) and (-2, 0)

Foci: (3.61, 0) and (-3.61, 0)

Graphing a Hyperbola

Graph: $\frac{(x+2)^2}{9} - \frac{(y-1)^2}{25} = 1$

Horizontal hyperbola

Center: (-2, 1)

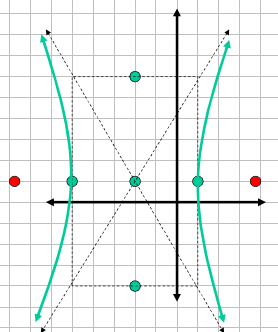
Vertices: (-5, 1) and (1, 1)

$$c^2 = 9 + 25 = 34$$

$$c = \sqrt{34} = 5.83$$

Foci: (-7.83, 1) and (3.83, 1)

Asymptotes: $y = \frac{5}{3}(x+2) + 1$
 $y = -\frac{5}{3}(x+2) + 1$



Converting an Equation

Graph: $9y^2 - 4x^2 - 18y + 24x - 63 = 0$

$$9(y^2 - 2y + \underline{1}) - 4(x^2 - 6x + \underline{9}) = 63 + \underline{9} - \underline{36}$$

$$9(y-1)^2 - 4(x-3)^2 = 36$$

$$\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1$$

The hyperbola is vertical

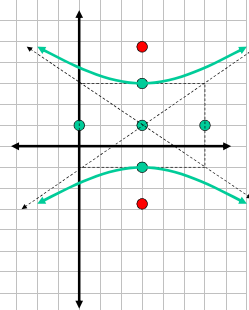
Center: (3, 1)

$$c^2 = 9 + 4 = 13$$

$$c = \sqrt{13} = 3.61$$

Foci: (3, 4.61) and (3, -2.61)

Asymptotes: $y = \frac{2}{3}(x-3) + 1$
 $y = -\frac{2}{3}(x-3) + 1$



Finding an Equation

Find the standard form of the equation of a hyperbola given:

Foci: (-7, 0) and (7, 0)

Vertices: (-5, 0) and (5, 0)

Horizontal hyperbola
Center: (0, 0)

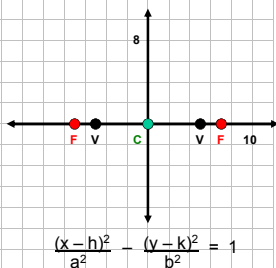
$$a^2 = 25 \text{ and } c^2 = 49$$

$$c^2 = a^2 + b^2$$

$$49 = 25 + b^2$$

$$b^2 = 24$$

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$



Finding an Equation

Find the standard form equation of the hyperbola that is graphed at the right

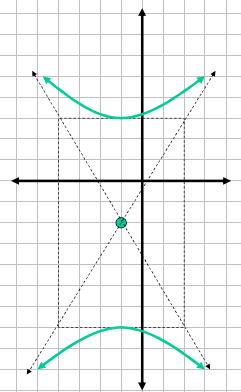
Vertical hyperbola

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Center: (-1, -2)

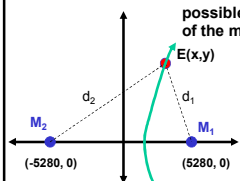
a = 5 and b = 3

$$\frac{(y+2)^2}{25} - \frac{(x+1)^2}{9} = 1$$



Applications

An explosion is recorded by two microphones that are two miles apart. M1 received the sound 4 seconds before M2, assuming that sound travels at 1100 ft/sec, determine the possible locations of the explosion relative to the locations of the microphones.



Let us begin by establishing a coordinate system with the origin midway between the microphones

Since the sound reached M2 4 seconds after it reached M1, the difference in the distances from the explosion to the two microphones must be $1100(4) = 4400$ ft wherever E is

This fits the definition of an hyperbola with foci at M1 and M2

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since $d_2 - d_1 =$ transverse axis, $a = 2200$

$$c^2 = a^2 + b^2$$

$$5280^2 = 2200^2 + b^2$$

$$b^2 = 23,038,400$$

The explosion must be on the hyperbola

$$\frac{x^2}{4,840,000} - \frac{y^2}{23,038,400} = 1$$