

Name: \_\_\_\_\_

### Algebra 2: Section 7-4, Ellipse

**Equations of Ellipses:** An **ellipse** is the set of all points in a plane such that the *sum* of the distances from two given points in the plane, called the **foci**, is constant. An ellipse has two axes of symmetry which contain the **major** and **minor axes**. In the table, the lengths  $a$ ,  $b$ , and  $c$  are related by the formula  $c^2 = a^2 - b^2$

\*\*\* Please note that  $a$ ,  $b$ , and  $c$  are related to the Pythagorean theorem, but not the same parts! \*\*\*

<b>Standard Form of Equation</b>	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where $b^2 = a^2 - c^2$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ where $b^2 = a^2 - c^2$
<b>Center</b>	<b>(h, k)</b>	<b>(h, k)</b>
<b>Direction of Major Axis</b>	<b>Horizontal</b>	<b>Vertical</b>
<b>Foci</b>	<b>(h - c, k) &amp; (h + c, k)</b>	<b>(h, k + c) &amp; (h, k - c)</b>
<b>Length of Major Axis</b>	<b>2a units</b>	<b>2a units</b>
<b>Length of Minor Axis</b>	<b>2b units</b>	<b>2b units</b>

Every ellipse has two axes of symmetry. The points at which the ellipse intersects the axes define two segments with endpoints on the ellipse. The longer segment is called the major axis; the shorter the minor axis. The foci always lie on the major axis. The intersection of the two axes creates the center.

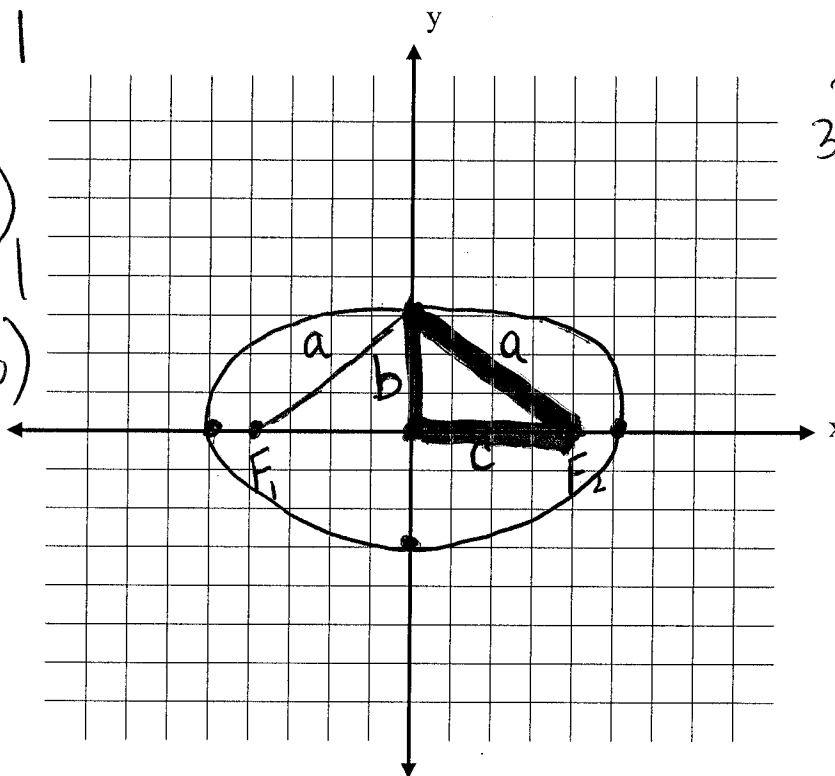
Eq:  $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Center (0,0)  
Horizontal

Foci (-4,0) & (4,0)

Major: 10

minor: 6



$$3^2 + 4^2 = a^2$$

$$\sqrt{25} = \sqrt{a^2}$$

$$5 = a$$

$$10 = 2a$$

The sum of the distances from the foci to any point on the ellipse is  $2a$ . The distance from the center to a focus is  $c$ . The length of the major axis is  $2a$ . The length of the minor axis is  $2b$ . Notice that  $a > b$ . It follows that  $a^2 > b^2$ . Thus, if  $a^2$  (the bigger number) is the denominator of the  $(x-h)^2$  term, the foci are on the horizontal  $x$ -axis. If  $a^2$  (the bigger number) is the denominator of the  $(y-k)^2$  term, the foci are on the vertical  $y$ -axis.

$$\text{Eq: } \frac{16x^2}{144} + \frac{4y^2}{144} = \frac{144}{144} \leftarrow \text{Need } 1$$

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

$$\frac{(x-0)^2}{9} + \frac{(y-0)^2}{36} = 1$$

Center:  $(0,0)$

$$\begin{array}{l} \rightarrow \\ b^2 \\ b=3 \end{array}$$

$$\begin{array}{l} \rightarrow \\ a^2 \\ a=6 \end{array}$$

Vertical

Major:  $2a \rightarrow 12$

Minor:  $2b \rightarrow 6$

$$\text{Foci: } b^2 = a^2 - c^2$$

$$9 = 36 - c^2$$

$$\begin{array}{r} -36 \\ \hline \sqrt{+27} = \sqrt{+c^2} \end{array}$$

$$\downarrow$$

$$\sqrt{9} \cdot \sqrt{3}$$

$$3\sqrt{3} = c$$

$$(h, k+c) \& (h, k-c)$$

$$(0, 3\sqrt{3}) \& (0, -3\sqrt{3})$$