

Section 5-9: Complex #'s

Challenge: Solve $3x^2 + 15 = 0$

$$\begin{array}{r} 3x^2 + 15 = 0 \\ -15 \quad -15 \\ \hline \end{array}$$

$$\frac{3x^2}{3} = \frac{-15}{3}$$

$$\sqrt{x^2} = \sqrt{-5}$$

$$x = \sqrt{-5}$$

↑
not a real root

Rene Descartes (Day-Cart)
Lived in the 1600's. He proposed
the number i , where i is not
a real number.

Definition $i = \sqrt{-1}$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$i^4 = 1$$

$$\sqrt{-1} = i$$

practice

$$\begin{array}{r}
 \text{a.) } \sqrt{-12} = \sqrt{-1} \sqrt{12} \\
 \quad \quad \quad \downarrow \quad \quad \downarrow \\
 \quad \quad \quad i \quad \quad \sqrt{4} \sqrt{3} \\
 \quad \quad \quad \quad \quad \downarrow \\
 \quad \quad \quad \quad \quad 2i\sqrt{3}
 \end{array}$$

i is "imaginary unit"
 $2i$ is called a "pure imaginary number"

$$\text{b.) } \sqrt{-27x^3}$$

$$\sqrt{-1} \sqrt{9} \sqrt{3} \sqrt{x^2} \sqrt{x}$$

$$i 3x \sqrt{3x}$$

$$3ix \sqrt{3x}$$

$$\begin{aligned}
 c.) \quad \sqrt{-81y^4} &= \sqrt{-1} \sqrt{81} \sqrt{y^4} \\
 &= \boxed{9iy^2}
 \end{aligned}$$

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$$\begin{aligned}
 a.) \quad -3i \cdot 8i &= -24i^2 \quad \text{substitute} \\
 &\quad \downarrow \\
 &= -24(-1) = \boxed{24}
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad \sqrt{-6} \cdot \sqrt{-10} &= i\sqrt{6} \cdot i\sqrt{10} \\
 &= i^2 \sqrt{60} \\
 &= i^2 \sqrt{4} \sqrt{15} \\
 &= 2i^2 \sqrt{15} \\
 &\quad \downarrow \\
 &= 2(-1) \sqrt{15} \\
 &= \boxed{-2\sqrt{15}}
 \end{aligned}$$

$$c.) \quad i^{25} = i^{2(12)} \cdot i = (-1)^{12} \cdot i = 1 \cdot i = i$$