Name: Key

Algebra 2 Crash Course: Solving Radical Equations

A circular shaped field has an area of 250,000 ft^2. What is the radius of this field? [Hint: A = πr^2]

$$\frac{250{,}000}{\pi} = \frac{\cancel{\pi} \cdot r^2}{\cancel{\pi}} \qquad r = 282 \text{ Feet}$$

Basic Ideas: $x^{1/2} = \sqrt{x}$ $\quad x^{1/3} = \sqrt[3]{x}$ $\quad x^{1/4} = \sqrt[4]{x}$

Basic Examples: Solve for x

$x^{1/2} = 81$

$$(\sqrt{x})^2 = (81)^2$$
$$x = 6561$$

$x^{1/3} = 2$

$$(\sqrt[3]{x})^3 = (2)^3$$
$$x = 8$$

$x^{1/4} = 5$

$$(\sqrt[4]{x})^4 = (5)^4$$
$$x = 625$$

Strategies for solving Radical Equations

<u>Strategy I</u>: Only one variable; not under radical

Isolate variable, divide everything except variable.

$x\sqrt{8} - 8 = 3$

$$\frac{x\cancel{\sqrt{8}}}{\cancel{\sqrt{8}}} = \frac{11}{\sqrt{8}}$$
$$x = \frac{11}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$$
$$= \frac{11\sqrt{8}}{8} = \frac{11 * 2\sqrt{2}}{8}$$
$$= \frac{22\sqrt{2}}{8}$$
$$= \frac{11\sqrt{2}}{4}$$

$1 + x\sqrt{2} = 0$

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$
$$x = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{-\sqrt{2}}{2}$$

$7 + 6x\sqrt{5} = 0$

$$\frac{6x\sqrt{5}}{6\sqrt{5}} = \frac{-7}{6\sqrt{5}}$$
$$x = \frac{-7}{6\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$x = \frac{-7\sqrt{5}}{30}$$

Strategy II: **Variable not under radical, but two variable terms present**

Get both variable terms on one side. Factor out the variable. Divide by everything but the variable.

$g\sqrt{7} + 8 = g$

$g\sqrt{7} - g = -8$

$\frac{g(\sqrt{7}-1)}{\sqrt{7}-1} = \frac{-8}{\sqrt{7}-1}$

$g = \frac{-8}{\sqrt{7}-1} \cdot \frac{\sqrt{7}+1}{\sqrt{7}+1} = \frac{-8\sqrt{7}-8}{7-1}$

$= \frac{-8\sqrt{7}-8}{6} = \frac{-4\sqrt{7}-4}{3}$

Conjugate

$y\sqrt{3} - y = 7$

$\frac{y(\sqrt{3}-1)}{\sqrt{3}-1} = \frac{7}{\sqrt{3}-1}$

$y = \frac{7}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$y = \frac{7\sqrt{3}+7}{3-1}$

$= \frac{7\sqrt{3}+7}{2}$

$13 - 3r = r\sqrt{5}$

$13 = r\sqrt{5} + 3r$

$\frac{13}{\sqrt{5}+3} = \frac{r(\sqrt{5}+3)}{\sqrt{5}+3}$

$\frac{13}{\sqrt{5}+3} * \frac{\sqrt{5}-3}{\sqrt{5}-3} = r$

$\frac{13\sqrt{5}-39}{5-9} = r$

$\frac{13\sqrt{5}-39}{-4} = r$

Strategy III: **Variable is under radical; only one variable term present**

Get radical containing variable on one side; everything else on the other side. Raise both sides to the power that will "undo" the radical's index.

$\sqrt[4]{7+3z} + 7 = 9$

$\left(\sqrt[4]{7+3z}\right)^4 = (2)^4$

$7 + 3z = 16$
$-7 \quad -7$

$\frac{3z}{3} = \frac{9}{3}$

$z = 3$

$\sqrt[3]{2y+1} = 3$

$\left(\sqrt[3]{2y+1}\right)^3 = (3)^3$

$2y + 1 = 27$
$-1 \quad -1$

$\frac{2y}{2} = \frac{26}{2}$

$y = 13$

$3 + \sqrt[2]{4n-5} = 10$
$-3 \quad -3$

$\left(\sqrt[2]{4n-5}\right)^2 = (7)^2$

$4n - 5 = 49$
$+5 \quad +5$

$\frac{4n}{4} = \frac{54}{4}$

$n = 13.5$

Strategy IV: **EVERYTHING is under radicals!**

Make sure one radical is on each side of equation. Raise both sides to the appropriate power to "undo" the radicals. Solve for x.

$\left(\sqrt{2m-6}\right)^2=\left(\sqrt{3+m}\right)^2$

$2m-6=3+m$

$-m \qquad -m$

$m-6=3$

$m=9$

$\left(\sqrt{g-4}\right)^2=\left(\sqrt{2g-3}\right)^2$

$g-4=2g-3$

$-1=g$

$\left(\sqrt{x-8}\right)^2=\left(\sqrt{13+x}\right)^2$

$x-8=13+x$

$0=21$

NO Solution

Strategy V: **Expressions under radical on both sides, plus other terms not under radical.**

Get *one* radical on *each* side, raise each side to the appropriate index power to "undo" radical. Expand and combine like terms. Get single radical by itself; raise both sides to power to undo the radical. Solve for x (~~probably~~ may need to factor)

~~$\left(1+\sqrt{x+5}\right)^2=\left(\sqrt{2x+5}\right)^2$~~

~~FOIL~~

~~$\left(1+\sqrt{x+5}\right)\left(1+\sqrt{x+5}\right)=2x+5$~~

~~$1+2\sqrt{x+5}+x+5=2x+5$~~

~~$2\sqrt{x+5}+x+6=2x+5$~~

~~$2\sqrt{x+5}=x-$ [illegible]~~

~~$\left(2\sqrt{x+5}\right)^2=\left(x-\text{[illegible]}\right)^2$~~

~~$4x+20=x^2-2x+1$~~

~~$0=x^2-6x-19$~~

$\left(\sqrt{a+1}\right)^2=\left(\sqrt{a+6}-1\right)\left(\sqrt{a+6}-1\right)$

$a+1=a+6-2\sqrt{a+6}+1$

$a+1=a+7-2\sqrt{a+6}$

$\dfrac{-6}{-2}=\dfrac{-2\sqrt{a+6}}{-2}$

$(3)^2=\left(\sqrt{a+6}\right)^2$

$9=a+6$

$\boxed{3=a}$

$\sqrt{h+3}+\sqrt{h-1}=5$

$\left(\sqrt{h+3}\right)^2=\left(5-\sqrt{h-1}\right)^2$

$h+3=25-10\sqrt{h-1}+h-1$

$h+3=24+h-10\sqrt{h-1}$

$\dfrac{-21}{-10}=\dfrac{-10\sqrt{h-1}}{-10}$

$\left(\dfrac{21}{10}\right)^2=\left(\sqrt{h-1}\right)^2$

$\dfrac{441}{100}=h-1$

$\dfrac{541}{100}=h$