Sec 5-5: Roots

radical sign

Index $\rightarrow \sqrt[3]{64}$

radicand

Note: $\sqrt{4}$ same as $\sqrt[2]{4}$

$\boxed{\text{Scenario 1}}$

Radical has even index, positive radicand

$$\sqrt[2]{36} = 6 \quad \underline{\text{OR}} \quad -6$$

principal root (positive)

$$\sqrt[4]{16} = 2 \quad \underline{\text{OR}} \quad -2$$

* get one positive, one negative root

$\boxed{\text{Scenario 2}}$ odd index, positive radicand

$$\sqrt[3]{27} = 3$$

* get one positive root only

Scenario 3 Even index, negative radicand

$$\sqrt[2]{-49} = ?$$

$$\sqrt[4]{-16} = ?$$

No Real solutions

Scenario 4 ODD index, negative radicand

$$\sqrt[5]{-32} = -2$$

* get one negative root

Scenario 5 $\sqrt{0} = 0$

Ex. A) $\sqrt{49x^8} = \boxed{7x^4}$

B) $-\sqrt[2]{(a^2+1)^4} = -(a^2+1)^2$

C.) $\sqrt[5]{32x^{10}y^{15}} = 2x^2y^3$

D.) $\sqrt{-16}$ = No Real solution

E.) $\sqrt{(-3)^2} = \sqrt{9} = \boxed{3}$

F.) $\sqrt[2]{(-2)^{10}} = \sqrt[2]{2^{10}} = 2^5 = \boxed{32}$

G.) $\sqrt[6]{x^6} = x$

H.) $\sqrt[4]{16(x+3)^{12}} = 2(x+3)^3$

I.) $\sqrt[3]{x^6 y^9 z^6 b^{30}} = x^2 y^3 z^2 b^{10}$

J.) $\sqrt[2]{(6x)^4} = (6x)^2 = 36x^2$

K.) $\sqrt{100y^6 z^{10}} = 10y^3 z^5$

L.) $\sqrt{x^2+8x+16} = x+4$

$\sqrt{(x+4)(x+4)}$

Section 5-5: Roots

INDEX $\sqrt[3]{64}$ radical sign

radicand

$$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$$

Examples:

even index, positive radicand ± 6

$$\sqrt[2]{36} = 6 \text{ or } -6$$

principal root

odd index, positive radicand

$$\sqrt[3]{64} = 4$$

only positive root

even index, negative radicand

$\sqrt[2]{-49}$

$\sqrt[4]{-16}$

impossible

* No Real Roots.

odd index, negative radicand

$\sqrt[5]{-32} = -2$

one negative root

$\sqrt{0} = 0$

a.) $\pm\sqrt{49x^8} = \pm 7x^4$

b.) $-\sqrt[2]{(a^2+1)^4} = -(a^2+1)^2$

$-\sqrt[2]{\left[(a^2+1)^2\right]^2}$

c. $\sqrt[5]{32x^{10}y^{15}} = \sqrt[5]{(2)^5(x^2)^5(y^3)^5}$

$= 2x^2y^3$

d. $\sqrt[2]{-16} \rightarrow$ no real solution

* When you find the root of an even power, and an odd power results, you must take the Absolute Value to ensure the value is non-negative.

a.) $\sqrt[2]{(-3)^2} = |-3| = 3$

b.) $\sqrt[2]{(-2)^{10}} = \sqrt[2]{[(-2)^5]^2} = |-2^5|$

$= 2^5 = 32$

c.) $\sqrt[6]{x^6} = |x^1| = |x|$

d.) $\sqrt[4]{16(x+3)^{12}}$

$\sqrt[4]{(2)^4\left[(x+3)^3\right]^4}$

$= 2\left|(x+3)^3\right|$

ex.) $\sqrt[3]{x^6 y^9 z^6 b^{30}}$

$\sqrt[3]{(x^2)^3 (y^3)^3 (z^2)^3 (b^{10})^3}$

$= x^2 y^3 z^2 b^{10}$

(49) $\sqrt[3]{-27a^9b^{12}}$

$\sqrt[3]{(-3)^3(a^3)^3(b^4)^3}$

$= -3a^3b^4$

(45.) $\sqrt[2]{36g^6} = 6|g^3|$

(35.) $\sqrt[4]{\left(-\frac{1}{2}\right)^4} = \left(-\frac{1}{2}\right)^1$

$\left|-\frac{1}{2}\right| = \frac{1}{2}$

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