

Sec 11-6: Recursion & Iterations

①

Look at a special Sequence

Fibonacci: 1, 1, 2, 3, 5, 8, ...

rule: each succeeding term is found by adding the previous term.

Formula:
$$a_n = a_{n-1} + a_{n-2}$$

Fibonacci sequence is an example of a recursive sequence.

A Recursive Formula has two parts:

- 1.) the value of the first term
- 2.) a recursion equation that shows how to find each term from the term or terms before it.

Ex 1). Find the first 5 terms
in the sequence when
 $a_1 = 5$ & $a_{n+1} = 2a_n + 3$

$$a_2 = 2(5) + 3 = 13$$

$$a_3 = 2(13) + 3 = 29$$

$$a_4 = 2(29) + 3 = 61$$

$$a_5 = 2(61) + 3 = 125$$

5, 13, 29, 61, 125

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(#4) $a_1 = 12, a_{n+1} = a_n - 3$

$$a_2 = 12 - 3 = 9$$

$$a_3 = 9 - 3 = 6$$

$$a_4 = 6 - 3 = 3$$

$$a_5 = 3 - 3 = 0$$

$$a_6 = 0 - 3 = -3$$

(3)

(#5) $a_1 = 1, a_2 = 2$; $a_{n+2} = 4a_{n+1} - 3a_n$

$a_3 = 5$	$a_4 = 14$
$a_5 = 41$	$a_6 = 122$

$$a_3 = 4(2) - 3(1) = 8 - 3 = 5$$

$$a_4 = 4(5) - 3(2) = 20 - 6 = 14$$

$$a_5 = 4(14) - 3(5) = 56 - 15 = 41$$

$$a_6 = 4(41) - 3(14) = 164 - 42 = 122$$

Consider this:

$$\sqrt{\sqrt{\sqrt{2}}}$$

$$\sqrt{2} = \sqrt{1.41} = \sqrt{1.19} = (1.09)$$

This is called an iteration because it's the process of composing a function with itself repeatedly.

(#6)

Find the first three iterations
of $f(x) = 3x - 4$, $x_0 = 3$
↖ initial value

$$f(3) = 3(3) - 4 = 5$$

$$f(5) = 3(5) - 4 = 11$$

$$f(11) = 3(11) - 4 = 29$$

} 3 iterations

(#8)

Find the first ~~five~~^{three} iterates
of $f(x) = 2x - 8$

$$x_0 = -7 \quad f(-7) = 2(-7) - 8 = -22$$

$$f(-22) = 2(-22) - 8 = -52$$

$$f(-52) = 2(-52) - 8 = -112$$

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