

# 11-4: Geometric Series ①

A Geometric Series is the sum of the terms of a Geometric Sequence.

A parental Family tree is a type of Geometric Series.

<u>Parents</u>	<u>Grandparents</u>	<u>Great-gp</u>	<u>GGGp</u>
2	4	8	16

$$S_4 = 2 + 4 + 8 + 16 = 30$$

We need a Formula for large values of  $n$ . Let's Explore!!!!

$$S_4 = 2 + 4 + 8 + 16$$

$$- 2S_4 = \quad 4 + 8 + 16 + 32$$

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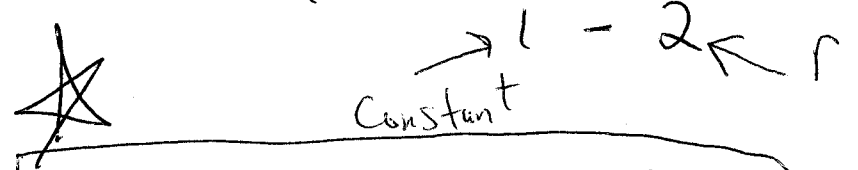
$$S_4 - 2S_4 = 2 + 0 + 0 + 0 - 32$$

$$S_4 \frac{(1-2)}{(1-2)} = \frac{2-32}{1-2} = 30$$

(2)

$$S_4 = 2 + 4 + 8 + 16$$

Analyse:  $S_4 = \frac{a_1 - a_1 r^n}{1 - r}$



So  $S_n = \frac{a_1 - a_1 r^n}{1 - r} \quad r \neq 1$

Ex1 Sum of first 6 terms  
 $a_1 = 5, r = -2$

$$S_6 = \frac{5 - 5(-2)^6}{1 - (-2)}$$

$$= \frac{5 - 5(64)}{3} = \frac{5 - 320}{3}$$

$$= \frac{-315}{3}$$

$S_6 = -105$

Ex2  $a_1 = 5, r = 2, n = 14$

$$S_{14} = \frac{5 - 5(2)^{14}}{1 - 2}$$

$$= \frac{5 - 5(16,384)}{-1} = \frac{5 - 81,920}{-1}$$

$$= \frac{-81,915}{-1}$$

$$S_{14} = 81,915$$

Sometimes we know the last term,  $a_n$ , but not how many terms in the series.

Alternate  
Geo  
Series

$$S_n = \frac{a_1 - a_n r}{1 - r}$$

Ex3 Find  $S_n$  when  $a_1 = 729$ ,  $a_n = -3$   
 $r = -\frac{1}{3}$

Alt. 
$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{729 - (-3)\left(-\frac{1}{3}\right)}{1 - -\frac{1}{3}}$$

$$= \frac{729 - 1}{\frac{4}{3}} = \frac{728}{\frac{4}{3}}$$

$$728 \div \frac{4}{3} \rightarrow \frac{728}{1} * \frac{3}{4} = \boxed{546}$$

Ex4 Find  $a_1$  when  $S_7 = 70,933$ ;  $r = 4$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$70,933 = \frac{a_1 - a_1(4)^7}{1-4}$$

$$(-3)70,933 = \frac{a_1 - a_1(16,384)}{-3}$$

$$-212799 = 1a_1 - a_1(16,384)$$

$$\frac{-212799}{(1-16,384)} = \frac{a_1(1-16,384)}{(1-16,384)}$$

$$12.989 = a_1$$

Ex 5

$$\sum_{n=1}^5 4(3)^{n-1}$$

Find Sum

$$4(3)^{1-1}$$

$$4(3)^0$$

$$4$$

$$4(3)^{2-1}$$

$$4(3)$$

$$12$$

$$a_1 = 4$$

$$n = 5$$

$$r = 3$$

$$S_5 = \frac{4 - 4(3)^5}{1-3}$$

$$= \frac{4 - 972}{-2} = 484$$

Ex 6 Find Sum:  $a_1 = 12$ ,  $r = -3$ ,  $a_5 = 972$

$$n = 5, a_1 = 12, r = -3$$

$$S_5 = \frac{12 - 12(-3)^5}{1 - -3}$$

$$= \frac{12 - -2916}{4} = 732$$

Ex 7 Find Sum  
3 - 6 + 12 ... to 7 terms

$$n = 7, r = -2, a_1 = 3$$

$$S_7 = \frac{3 - 3(-2)^7}{1 - -2}$$

$$= \frac{3 - -384}{3} = 129$$