Name: Key 3/9/2020

HansenMath Pre-calc: 7.8 Applications of Matrices and Determinants

Area of a Triangle

In this section, you will study some additional applications of matrices and determinants. The first involves a formula for finding the area of a triangle whose vertices are given by three points on a rectangular coordinate system.

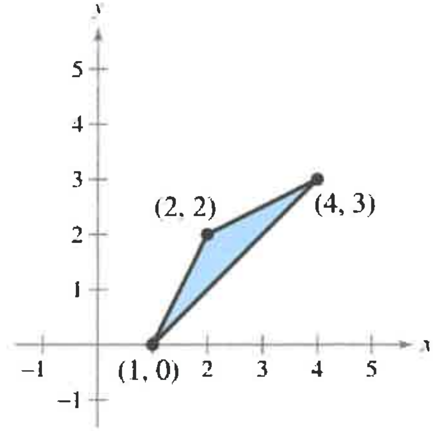


Figure 7.34

> **Area of a Triangle**
>
> The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is
>
> $$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$
>
> where the symbol $(\pm)$ indicates that the appropriate sign should be chosen to yield a positive area.

Example 1 Finding the Area of a Triangle

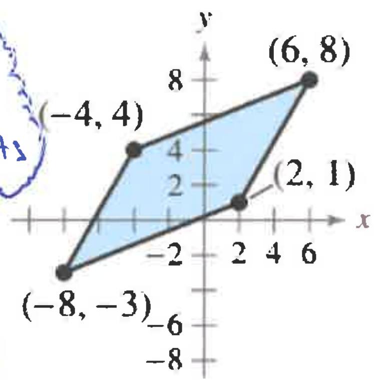
Find the area of the triangle whose vertices are (1, 0), (2, 2), and (4, 3), as shown in Figure 7.34.

$$\text{Area} = \pm\frac{1}{2}\underbrace{\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix}}_{[A]}$$

$$= \pm\frac{1}{2} * \det([A]) = -\frac{1}{2} \cdot -3 = 1.5$$

sq. units

Example 2: Use logic and the formula above to find the area of this parallelogram.



* Choose 3 points as that gives a Triangle. Since there's 2 Triangles don't use ½ in Formula.

$$\text{Area} = \pm\underbrace{\begin{vmatrix} 6 & 8 & 1 \\ 2 & 1 & 1 \\ -4 & 4 & 1 \end{vmatrix}}_{[A]}$$

$$\text{Area} = \pm\det[A] = \boxed{54 \text{ units}^2}$$

Example 3: Suppose three given points are (0, 1) (2, 2) and (4, 3). What would the area be of the "triangle"?

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} \leftarrow [A]$$

$$\text{Area} = \pm\frac{1}{2} * \det([A]) = 0$$

No Area! 0 Tells you points are collinear!!

Example 4: Are the given three points collinear? (-2, -2) (1, 1) and (7, 5)

Test to see if det is 0.

$$[A] = \begin{vmatrix} -2 & -2 & 1 \\ 1 & 1 & 1 \\ 7 & 5 & 1 \end{vmatrix} \rightarrow \det([A]) = -6$$

So, Not collinear!

Cryptography

A **cryptogram** is a message written according to a secret code. (The Greek word *kryptos* means "hidden.") Matrix multiplication can be used to encode and decode messages. To begin, you need to assign a number to each letter in the alphabet (with 0 assigned to a blank space), as follows.

0 = _	9 = I	18 = R
1 = A	10 = J	19 = S
2 = B	11 = K	20 = T
3 = C	12 = L	21 = U
4 = D	13 = M	22 = V
5 = E	14 = N	23 = W
6 = F	15 = O	24 = X
7 = G	16 = P	25 = Y
8 = H	17 = Q	26 = Z

To encode a secret message

Real Word Matrix (1 x 3)		Encoding Matrix (3 x 3)		Secret-code Word Matrix (3 x 1)
[A]	*	[B]	=	[C]

To decipher and crack the code, it follows...

Secret-code Word Matrix (3 x 1)		Inverse Encoding Matrix (3 x 3)		Decoded Real Word Matrix (1 x 3)
[C]	*	$[B^{-1}]$	=	[A]

[Note: The above conveys the general idea, but you will use different Matrix letters to suit your needs]

*OR just use one 4x3 matrix!

Example 5: Decode the cryptogram given by: 4, -24, 83, 16, -35, 46, 40, -65, -10, 0, -6, 30

If the encoding matrix is $\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$

I set as [E] in calc.

Use Graphing Utility

Set: [A] = [4 -24 83]
[B] = [16 -35 46]
[C] = [40 -65 -10]
[D] = [0 -6 30]

So... $[A]*[E]^{-1} = [20\ 21\ 5]$
$[B]*[E]^{-1} = [19\ 4\ 1]$
$[C]*[E]^{-1} = [25\ 0\ 15]$
$[D]*[E]^{-1} = [6\ 6\ 0]$

Use Chart ↑ : "Tuesday OFF"

And that's all folks ☺ It's time for the Chapter 7 Review Study Guide!