Name: 3/4/2020

HansenMath[™] Pre-calc: 7.6 Inverse of a Square Matrix

Do you remember the definition of the multiplicative identity property? It simply means that any time you multiply a number by ____, the result is the original number. In other words, it preserves the original number's identity! For example, 23 x = 23. See, the number 23 gets to keep its identity ©

With Matrices, the idea is very similar. However, instead of multiplying a matrix by a scalar of 1, you can multiply by what's called an identity matrix. This is a square matrix consisting of 1s on the diagonal and zeroes elsewhere. Let's take a look ©

Example 1: Multiply a Matrix by its identity Matrix

 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{What is A*I?} \qquad 0 + 2 \qquad 0 + 4 \qquad 0 + 4$

Next, remember that a multiplicative inverse is the number you multiply to another number to get __

For example, 8 * $\mathcal{F} = \mathcal{F}$

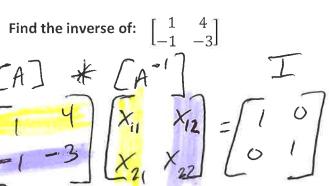
With Matrices, the idea is very similar. If you multiply a matrix A, by its inverse A⁻¹, you get

Example 2: Multiply to get the product, AB. $R = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \rightarrow R_2 \begin{bmatrix} -1+1 & 2-1 \\ 1 & 2-1 \end{bmatrix} \rightarrow 0$ I dentity

What must be true of Matrix A and B then? In verses of each other.

But how do we find an inverse in the first place???

If a matrix A has an inverse, A is called invertible (or nonsingular); otherwise, A is called singular. A non-square matrix cannot have an inverse. Not all square matrices have inverses. If a matrix does have an inverse, that inverse is unique.



Finding an Inverse Matrix

Let A be a square matrix of order n.

- 1. Write the $n \times 2n$ matrix that consists of the given matrix A on the left and the $n \times n$ identity matrix I on the right to obtain [A : I].
- 2. If possible, row reduce A to I using elementary row operations on the entire matrix [A : I]. The result will be the matrix $[I : A^{-1}]$. If this is not possible, A is not invertible.
- 3. Check your work by multiplying to see that $AA^{-1} = I = A^{-1}A$.

$$\begin{bmatrix} x_{11} + 4x_{21} & x_{12} + 4x_{22} \\ -x_{11} - 3x_{21} & -x_{12} - 3x_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{vmatrix}
-X_{11} - 3X_{21} - X_{12}^{-3}X_{22} \\
X_{11} + 4X_{21} = 1
\end{vmatrix} \begin{cases}
X_{21} = 1 \\
X_{21} = 1
\end{cases}$$

$$X_{12} + 4X_{22} = 0$$

$$X_{22} = 1$$

$$-X_{11} - 3X_{21} = 0$$

$$X_{12} = 3X_{22} = 1$$

$$X_{12} = -4$$

Example 4: Finding the Inverse of a Matrix, take two!

Find the inverse of:
$$\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & \vdots & 1 & 6 \\ -1 & -3 & \vdots & 0 & 1 \\ 2 \times 4 & & & & \end{bmatrix}$$

Calc - Matrix -> [A]

RREF Inverse

Example 5: Find the Inverse - BUT this time use your calculator!

Find the inverse of
$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$
. Edit Matrix A

There is a find the inverse of $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$.

$$3 \times 3$$

Example 6: Find the Inverse - use your calculator again!

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix} \quad \text{No Inverse}$$



$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then A is invertible if and only if $ad - bc \neq 0$. If $ad - bc \neq 0$, the inverse is given by

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Formula for inverse of matrix A

Example 7: Use the formula above to find the inverse of the 2x2 matrix.

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \\ c & d \end{bmatrix}.$$

$$A^{-1} = \frac{1}{6-2}\begin{bmatrix} 2\\2\\3 \end{bmatrix} \rightarrow \begin{bmatrix} 4\\2\\3 \end{bmatrix}$$

Example 8: Use an inverse Matrix to solve the system! (An alternative to RREF)

$$\begin{cases} 2x + 3y + z = -1 \\ 3x + 3y + z = 1 \\ 2x + 4y + z = -2 \end{cases}$$

Assignment: Page 547: #1, 3, 5 (Multiply, by hand)

#11, 13, 15 (by hand, any method from these notes)

#21, 25, 51, 55, 75 (Use calculator. Use any method you know to solve)