Name: 3/2/2020

HansenMath™ Pre-calc: 7.5 Operations with Matrices, Day 2

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ matrices and let c and d be scalars.

1. $A + B = B + A$ — Commutative Property of Matrix Addition
2. $A + (B + C) = (A + B) + C$ — Associative Property of Matrix Addition
3. $(cd)A = c(dA)$ — Associative Property of Scalar Multiplication
4. $1A = A$ — Scalar Identity
5. $A + O = A$ — Additive Identity
6. $c(A + B) = cA + cB$ — Distributive Property
7. $(c + d)A = cA + dA$ — Distributive Property

Representation of Matrices

1. A matrix can be denoted by an uppercase letter such as A, B, or C.
2. A matrix can be denoted by a representative element enclosed in brackets, such as $[a_{ij}]$, $[b_{ij}]$, or $[c_{ij}]$.
3. A matrix can be denoted by a rectangular array of numbers such as

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}.$$

Example 1: Solve for x and y

$$\begin{bmatrix} -5 & x \\ y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$$

match positions

$x = 13$

$y = 12$

Example 2: Find A+B, 5A, and 5A - B

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$$

ADD like elements; distribute scalars

$$A+B = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix} \qquad 5A = \begin{bmatrix} 5 & 10 \\ 10 & 5 \end{bmatrix}$$

$$5A - B = \begin{bmatrix} 5-(-3) & 10-(-2) \\ 10-4 & 5-2 \end{bmatrix} \rightarrow \begin{bmatrix} 8 & 12 \\ 6 & 3 \end{bmatrix}$$

Example 3: Find A+B

$$A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$$

2×3 $\qquad$ 2×2

* NOT defined; must have same dimensions to ADD

Example 4: Use your Graphing Calculator to solve.

$$-1\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \frac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right)$$

[A] [B] [C]

$$-1*[A] + \left(\tfrac{1}{6}\right)([B] + [C]) = \begin{bmatrix} -\frac{11}{3} & -\frac{3i}{3} \\ 1 & \frac{3}{2} \\ -8 & -1 \end{bmatrix}$$

Assignment: Page 536, #1, 3, 7, 12, 16, 17, 19, 23. That's all, folks ☺