Name: 2/28/2020

HansenMath™ Pre-calc: 7.5 Operations with Matrices, Day 1

Matrix Multiplication

Another basic matrix operation is **matrix multiplication.** At first glance, the following definition may seem unusual. You will see later, however, that this definition of the product of two matrices has many practical applications.

> **Definition of Matrix Multiplication**
>
> If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix given by
>
> $AB = [c_{ij}]$
>
> where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj}$.

Be sure you understand that for the product of two matrices to be defined, the number of *columns* of the first matrix must equal the number of *rows* of the second matrix. That is, the middle two indices must be the same. The outside two indices give the order of the product, as shown in the following diagram.

$$\underset{m \times n}{A} \times \underset{n \times p}{B} = \underset{m \times p}{AB}$$

Equal

Order of AB

Example 1: Multiply

$$\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1(2)+(-2)(-1)+(-3)(1) \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

$1\times 3 \quad 3\times 1$ yields 1×1

Example 2: Multiply

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} =$$

$\underbrace{3\times 1 \quad 1\times 3}$ yields 3×3

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ R_1 & 2(1) & 2(-2) & 2(-3) \\ R_2 & -1(1) & -1(-2) & -1(-3) \\ R_3 & 1(1) & 1(-2) & 1(-3) \end{array} \rightarrow \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$$

Example 3:

Find the product AB using $A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$.

$\underbrace{3\times 2 \qquad 2\times 2}$

$$\begin{array}{c|cc} & C_1 & C_2 \\ R_1 & (-1)(-3)+3(-4) & -1(2)+3(1) \\ R_2 & 4(-3)+(-2)(-4) & 4(2)+(-2)(1) \\ R_3 & 5(-3)+(0)(-4) & 5(2)+(0)(1) \end{array}$$

Answer

$$\begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$$

Example 4: Multiply

a. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} =$

$\underbrace{2\times 3 \qquad 3\times 3}$

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ R_1 & 1(-2)+0+3 & 4+0+3 & 2+0-3 \\ R_2 & -4+-1+2 & 8+0+-2 & 4+0+2 \end{array} \rightarrow \begin{bmatrix} -5 & 7 & -1 \\ -3 & 6 & 6 \end{bmatrix}$$

Example 4, Continued:

b. $\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

2×2 2×2

$$\begin{array}{c|cc} & C_1 & C_2 \\ R_1 & 3+0 & 0+4 \\ R_2 & -2+0 & 0+5 \end{array} \rightarrow \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

c. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$

$$\begin{array}{c|cc} & C_1 & C_2 \\ R_1 & -1+2 & 2+-2 \\ R_2 & -1+1 & 2+-1 \end{array} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

f. $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$

3×2 3×4

mismatch → NOT possible: AB

Example 5:

Two softball teams submit equipment lists to their sponsors, as shown in the table at the right. Each bat costs \$80, each ball costs \$6, and each glove costs \$60. Use matrices to find the total cost of equipment for each team.

Equipment	Women's Team	Men's Team
Bats	12	15
Balls	45	38
Gloves	15	17

$$\begin{bmatrix} 80 & 6 & 60 \end{bmatrix} \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} \quad \begin{array}{cc} W & M \\ \end{array} \begin{bmatrix} \$2130 & \$2448 \end{bmatrix}$$

1 × 3 3 × 2 → 1 × 2

<u>Assignment</u>: **Page 537, #25-37odd (by hand)**
#41, 45, 48 (use calculator to multiply)
#57, 58 (use calculator RREF)
#79, 80, 81 (setup then solve w/ calculator)
Catch 'Em All. Thank you ☺