Name: 2/24/20

HansenMath™ Pre-calc: 7.4 Matrices and Systems of Equations, Day 2

Example 1: What do you call the form of this matrix?

Augmented - reduced row-echelon

What does it tell you?

$$\begin{array}{cc} \left[\begin{array}{ccc:c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 2 \end{array}\right] & \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

$(-4, -8, 2)$

Gaussian Elimination with Back-Substitution

1. Write the augmented matrix of the system of linear equations.
2. Use elementary row operations to rewrite the augmented matrix in row-echelon form.
3. Write the system of linear equations corresponding to the matrix in row-echelon form and use back-substitution to find the solution.

Example 2: Write the system as an augmented matrix. Solve via Gaussian Elimination.

$$\begin{cases} x + 2y + z = -4 \\ 2x - y + z = -4 \\ x + 3y - z = -7 \end{cases}$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & -4 \\ 2 & -1 & 1 & -4 \\ 1 & 3 & -1 & -7 \end{array}\right] \begin{array}{c} (-2)R_1 \rightarrow \\ (-1)R_1 \rightarrow \end{array} \left[\begin{array}{cccc} 1 & 2 & 1 & -4 \\ 0 & -5 & -1 & 4 \\ 0 & 1 & -2 & -3 \end{array}\right]$$

$$(5)R_3 \rightarrow \left[\begin{array}{cccc} 1 & 2 & 1 & -4 \\ 0 & -5 & -1 & 4 \\ 0 & 0 & -11 & -11 \end{array}\right] \xrightarrow{\left(-\frac{1}{11}\right)R_3} \left[\begin{array}{cccc} 1 & 2 & 1 & -4 \\ 0 & -5 & -1 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

so $z = 1$, etc...

Gauss–Jordan Elimination

With Gaussian elimination, elementary row operations are applied to a matrix to obtain a (row-equivalent) row-echelon form of the matrix. A second method of elimination, called **Gauss-Jordan elimination** after Carl Friedrich Gauss (1777–1855) and Wilhelm Jordan (1842–1899), continues the reduction process until a *reduced* row-echelon form is obtained.

This Gauss-Jordan method is great because there's no back substitution. Once the matrix is *reduced* row-echelon, the solutions are staring you right in the face. Just like example 1 above, right?

But this is a **lot** of work - with lots of opportunity for error. Hey, it's 2020, we've got technology. We've got this! RREF to the rescue...

* Instructions for using your TI-84 calculator to write a matrix and solve to reduced row-echelon form *

Step 1: Press 2nd MATRIX → → **EDIT enter**. This will edit matrix [A].
Step 2: Enter the order, or the dimensions: m **enter** x n **enter**.
Step 3: Next, type in the values. You may hit **enter** after each entry to move to the right by row.
When finished, press 2nd QUIT
Step 4: Press 2nd MATRIX → **MATH.** Cursor up or down until you see **B: RREF**. **enter**.
Step 5: Press 2nd MATRIX **enter enter** to perform RREF on Matrix [A]. You'll get your solution in RREF form ☺

Example 2 (too good to be true?): Use your calculator to solve the system as an augmented matrix, using RREF

$$\begin{cases} x + 2y + z = -4 \\ 2x - y + z = -4 \\ x + 3y - z = -7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -4 \\ 2 & -1 & 1 & -4 \\ 1 & 3 & -1 & -7 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

3×4

$(-3, -1, 1)$

Example 3 (how sweet can it be?): Use your calculator to solve the system as an augmented matrix, using RREF

$$\begin{cases} x - 3z = -5 \\ 3x + y - 2z = -4 \\ 2x + 2y + z = -2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 3 & 1 & -2 & -4 \\ 2 & 2 & 1 & -2 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$(1, -3, 2)$

Example 4 (who could ask for more?): Use calculator to solve the system as an augmented matrix, using RREF

$$\begin{cases} x + y - 5z = 3 \\ x - 2z = 1 \\ 2x - y - z = 0 \end{cases}$$

$$\xrightarrow{\text{RREF}} \left[\begin{array}{cccc} 1 & 0 & -2 & 1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 = & 0 \end{array}\right] \rightarrow y - 3a = 2$$

$\boxed{y = 3a + 2}$

Infinite Sdn

R_3 All zero

let $\boxed{z = a}$

$x - 2a = 1$

$\boxed{x = 2a + 1}$

Assignment: **Page 522, #48-50 (by hand), then use calc: #55-59, 73, 79, 84. Try 'Em. Thank you ☺**