Name: 2/21/2020

HansenMath™ Pre-calc: 7.4 Matrices and Systems of Equations, Day 1

A Matrix is a rectangular array of real numbers.

The plural is matrices. These are used to help us solve systems of linear equations.

Definition of Matrix

If m and n are positive integers, an $m \times n$ (read "m by n") matrix is a rectangular array

$$\begin{array}{c} \\ \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \vdots \\ \text{Row } m \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Column 1} & \text{Column 2} & \text{Column 3} & \cdots & \text{Column } n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

in which each **entry** a_{ij} of the matrix is a real number. An $m \times n$ matrix has m rows and n columns.

A matrix having m rows and n columns is said to have be of Order $m \times n$. This can also be called its *dimensions*. If $m = n$ the matrix is **square** of order n.

Example 1 Order of Matrices

Determine the order of each matrix.

a. $[2]$ **b.** $\begin{bmatrix} 1 & -3 & 0 & \frac{1}{2} \end{bmatrix}$ **c.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **d.** $\begin{bmatrix} 5 & 0 \\ 2 & -2 \\ -7 & 4 \end{bmatrix}$

1x1 1x4 2x2 3x2

A matrix derived from a system of linear equations (each written in standard form with the constant term on the right) is the augmented matrix of the system.
Moreover, the matrix derived from the coefficients of the system is the Coefficient matrix of the system.

System
$$\begin{cases} x - 4y + 3z = 5 \\ -x + 3y - z = -3 \\ 2x - 4z = 6 \end{cases}$$

Augmented Matrix
$$\left[\begin{array}{ccc:c} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{array}\right]$$

Coefficient Matrix
$$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$$

Example 2 Writing an Augmented Matrix

Write the augmented matrix for the system of linear equations.

$$\begin{cases} x + 3y = 9 \\ -y + 4z = -2 \\ x - 5z = 0 \end{cases} \qquad \left[\begin{array}{ccc:c} 1 & 3 & 0 & 9 \\ 0 & -1 & 4 & -2 \\ 1 & 0 & -5 & 0 \end{array}\right]$$

In Section 7.3, we studied three operations that can be used on a system of linear equations to produce an equivalent system. We can also apply these operations in Matrix form.

Elementary Row Operations for Matrices

1. Interchange two rows.
2. Multiply a row by a nonzero constant.
3. Add a multiple of a row to another row.

Row-Echelon Form and Reduced Row-Echelon Form

A matrix in **row-echelon form** has the following properties.

1. Any rows consisting entirely of zeros occur at the bottom of the matrix.
2. For each row that does not consist entirely of zeros, the first nonzero entry is 1 (called a **leading 1**).
3. For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

A matrix in *row-echelon form* is in **reduced row-echelon form** if every column that has a leading 1 has zeros in every position above and below its leading 1.

Example 3: **Use row operations to re-write the system as a matrix in row-echelon form.**

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y + z = -2 \\ 2x - 5y + 5z = 17 \end{cases} \qquad \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc:c} 1 & -2 & 3 & 9 \\ -1 & 3 & 1 & -2 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

$$\begin{array}{c} \\ R_1+R_2 \\ \\ \end{array}\left[\begin{array}{ccc:c} 1 & -2 & 3 & 9 \\ 0 & 1 & 4 & 7 \\ 2 & -5 & 5 & 17 \end{array}\right] \qquad \left[\begin{array}{ccc:c} 1 & -2 & 3 & 9 \\ 0 & 1 & 4 & 7 \\ 0 & -1 & -1 & -1 \end{array}\right] R2+R3 \qquad \left[\begin{array}{ccc:c} 1 & -2 & 3 & 9 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 3 & 6 \end{array}\right] \leftarrow \frac{1}{3}$$

$(-2)R1: -2 \quad 4 \quad -6 \quad -18$

$$* \left[\begin{array}{ccc:c} 1 & -2 & 3 & 9 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Example 5 Row-Echelon Form

Determine whether each matrix is in row-echelon form. If it is, determine whether the matrix is in reduced row-echelon form.

a. $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 \end{bmatrix}$

c. $\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ **d.** $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$ **f.** $\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Row-echelon	reduced
a	d
c	F
d F	

Assignment: **Page 521, #1-17EOO, 23-28, 29, 34-36, 42, 44 Thank you ☺**