

Name: 2/19/20

HansenMath™ Pre-calc: 7.3 Multivariable Linear Systems, Day 2

Warm-up:

A system of linear equations is called consistent if it has at least one solution.

A consistent system with exactly one solution is independent. A consistent system with infinitely many solutions is *dependent*. A system of linear equations is called *inconsistent* if it has no solution.

Example 1: (No solution; Inconsistent)

Solve the system of linear equations:

$$\begin{cases} x - 3y + z = 1 \\ 2x - y - 2z = 2 \\ x + 2y - 3z = -1 \end{cases}$$

$$\begin{array}{l} [-2] \text{EQ1: } -2x + 6y - 2z = -2 \\ \text{EQ2: } 2x - y - 2z = 2 \\ \hline 5y - 4z = 0 \text{ New EQ2} \end{array}$$

$$\begin{array}{l} [-1] \text{EQ1: } -x + 3y - z = -1 \\ \text{EQ3: } x + 2y - 3z = -1 \\ \hline 5y - 4z = -2 \end{array}$$

$$\begin{array}{l} -5y + 4z = 0 \\ 5y - 4z = -2 \\ \hline * \boxed{0 = -2} \end{array}$$

No soln - inconsistent

Example 2: (Infinitely Many Solutions)

$$\begin{cases} x + 0y + 4z = 13 \\ 4x - 2y + z = 7 \\ 2x - 2y - 7z = -19 \end{cases}$$

$$(-4)EQ1: -4x + 0y - 16z = -52$$

$$EQ2: 4x - 2y + z = 7$$

$$\hline -2y - 15z = -45$$

$$(-2)EQ1: -2x + 0y - 8z = -26$$

$$EQ3: 2x - 2y - 7z = -19$$

$$\hline -2y - 15z = -45$$

$$x = -4a + 13$$

$$-2y - 15(a) = -45$$

$$\frac{-2y}{-2} = \frac{15a - 45}{-2} \rightarrow y = \frac{-15a + 45}{2}$$

$$y = \frac{-15a + 45}{2}$$

$$z = a$$

any $\mathbb{R} \#$

$0 = 0$ infinite solutions

Example 3: (A non-square system; few equations than variables)

$$\begin{cases} x - 2y + z = 2 \\ 6x - 3y - 3z = 3 \\ 2x - y - z = 1 \end{cases}$$

$$\left(-4a + 13, \frac{-15a + 45}{2}, a \right)$$

$$(2x - y - z = 1)$$

$$(-2)EQ1: -2x + 4y - 2z = -4$$

$$EQ2: 2x - y - z = 1$$

$$\hline 3y - 3z = -3$$

$$\begin{aligned} x - 2(z-1) + z &= 2 \\ x - 2z + 2 + z &= 2 \\ x - z + 2 &= 2 \\ x - z &= 0 \\ x &= z \end{aligned}$$

Solve for y

$$\frac{3y}{3} = \frac{3z - 3}{3}$$

$$y = z - 1$$

Into EQ 1

Let $z = a$

$$a = 2 \left(\frac{a}{2}, a - 1, a \right)$$