Name: Key 2/18/2020

HansenMath™ Pre-calc: 7.3 Multivariable Linear Systems, Day 1

The method of elimination can be applied to a system of linear equations in more than two variables. When elimination is used to solve a system of linear equations, the goal is to rewrite the system in a form to which back-substitution can be applied.

Example 1 below is said to be in **row-echelon form**, which means that it has a "stair-step" pattern with leading coefficients of 1. It is easier to solve the system in row-echelon form, using back-substitution.

Example 1

Row-echelon form

Solve the system of linear equations:

$$\begin{cases} x + y - z = 9 \\ \quad y - 2z = 4 \\ \qquad z = 1 \end{cases}$$

$y - 2(1) = 4$

$y = 6$

$x + y - z = 9$

$x + 6 - 1 = 9$

$x + 5 = 9$

$x = 4$

Write as "ordered triple": $(\underset{x}{4}, \underset{y}{6}, \underset{z}{1})$

To solve a system that is not is row-echelon form, Convert it to an equivalent system that IS in row-echelon form.

This process is called Gaussian Elimination named after the famous German mathematician, Carl Friedrich Gauss (1777 - 1855)

There are three elementary row operations that can be used on a system of linear equations to produce an equivalent system of linear equations:

1.) Interchange two equations

2.) Multiply one of the equations by a non-zero constant

3.) Add a multiple of one equation to another equation.

The number of solutions of a system of linear equations in more than two variables is:

- exactly one solution
- Infinitely many solutions
- No solution

Example 2: Solve the system

$\boxed{\text{STEP 1}}$ eliminate x from EQ2

$$\begin{cases} x + y + z = 3 & (1) \\ 2x - y + 3z = 16 & (2) \\ x - 2y - z = 1 & (3) \end{cases}$$

$\to$ (-2)EQ1: $-2x-2y-2z=-6$
EQ2: $2x-y+3z=16$

$-3y+z=10$
New EQ2

(1) $x+y+z=3$
(2) $-3y+z=10$
(3) $x-2y-z=1$

$\boxed{\text{STEP 2}}$ eliminate X From EQ3 $\to$ EQ1: $x+y+z=3$
(-1)EQ3: $-x+2y+z=-1$

$3y+2z=2$
New EQ3

(1) $x+y+z=3$
(2) $-3y+z=10$
(3) $3y+2z=2$

$\boxed{\text{STEP 3}}$
eliminate y from EQ3
EQ2: $-3y+z=10$
EQ3: $3y+2z=2$

$3z=12$
$z=4$
So... Back Sub
$(1, -2, 4)$

Example 3: Solve the system

$$\begin{cases} x - 2y + 3z = 9 & (1) \\ -x + 3y + z = -2 & (2) \\ 2x - 5y + 5z = 17 & (3) \end{cases}$$

$\boxed{\text{STEP 1:}}$ ADD (1) & (2) Eliminate X From EQ2
EQ1: $x-2y+3z=9$
EQ2: $-x+3y+z=-2$

$y+4z=7$ *new EQ2

(1) $x-2y+3z=9$
(2) $y+4z=7$
(3) $2x-5y+5z=17$

$\boxed{\text{STEP 2:}}$ Eliminate x from EQ3
-2(EQ1): $-2x+4y-6z=-18$
EQ 3: $2x-5y+5z=17$

$-y-z=-1$
*new EQ 3

(1) $x-2y+3z=9$
$y+4z=7$
$-y-z=-1$

$\boxed{\text{STEP 3:}}$ Eliminate y from EQ3
EQ2: $y+4z=7$
EQ3: $-y-z=-1$

$3z=6 \to z=2$
Back Sub, so...
$(1, -1, 2)$

Assignment: Page 505, #5, 7, 9, 13, 16, 17, 19, 27, 28 Thank you ☺