Name: 2/17/20

HansenMath™ Pre-calc: 7.2 Systems of Linear Equations in Two Variables

Story: Mr. Hansen has opened (hypothetically, of course) a food truck. He is selling Ribeye steaks for $20 each and his special-recipe Black-Bean-Hummus bowls (I love my vegan students, too) for $8. If Mr. Hansen collects $580 from selling 38 entrees after his first weekend of sales, how many of each entree did he sell? How do you solve this?? Systems of equations are invaluable for solving this and similar problems!

$\boxed{\$20r + \$8h = \$580}$

$-8\,[\,r + h = 38\,]$

$\boxed{-8r - 8h = -304}$

$\frac{12r}{12} = \frac{276}{12}$

$r = 23$ ribeyes

$h = 15$ hummus

> **The Method of Elimination**
>
> To use the method of elimination to solve a system of two linear equations in x and y, perform the following steps.
>
> 1. Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
> 2. Add the equations to eliminate one variable; solve the resulting equation.
> 3. Back-substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.
> 4. Check your solution in both of the original equations.

Example 1 (opposite # coefficients):

Solve the system of linear equations:

3x + 2y = 4

5x - 2y = 8

$\frac{8x}{8} = \frac{12}{8}$

$x = 1.5$

$3x + 2y = 4$

$3(1.5) + 2y = 4$

$4.5 + 2y = 4$

$-4.5 \qquad -4.5$

$2y = -.5$

$y = -.25$

$* \left(1.5, -.25\right)$ $\left(\frac{3}{2}, -\frac{1}{4}\right)$

Example 2 (non-opposite # coefficients):

Solve the system of linear equations:

$4\,[\,5x + 3y = 9\,]$

$3\,[\,2x - 4y = 14\,]$

$20x + 12y = 36$

$6x - 12y = 42$

$26x = 78$

$x = 3$

Then: $5(3) + 3y = 9$

$15 + 3y = 9$

$y = -2$

$* (3, -2)$

Example 3 (fraction style):

Solve the system of linear equations:

$6\left[\frac{x-1}{2} + \frac{y+2}{3} = 4\right]$ and $\boxed{x - 2y = 5}$

$3(x-1) + 2(y+2) = 24$

$3x - 3 + 2y + 4 = 24$

$\boxed{3x + 2y = 23}$

$x - 2y = 5$

$4x = 28 \rightarrow x = 7$

Example 4 (no solution / inconsistent / parallel):

$12\left[\frac{1}{4}x+\frac{1}{6}y=1\right]$ and $\boxed{-3x - 2y = 0}$

$3x+2y=12$

$-3x-2y=0$

$0=12$

No Solution (Parallel)

Example 5 (Infinitely many solutions / same line):

$-2\left[2x - y = 1\right]$ $\quad -4x+2y=-2$

$4x - 2y = 2 \rightarrow \quad 4x-2y=2$

$0=0$

True

Infinitely Many Soln (Same line)

Example 6 (Using your graphing calculator to solve)

2x + y = 5 and x - 2y = -1

$-2x \quad -2x \qquad -x \quad -x$

$y_1=-2x+5 \qquad \frac{-2y}{-2}=\frac{-x}{-2}\frac{-1}{-2}$

$y_2=\frac{1}{2}x+\frac{1}{2}$

(1.8, 1.4)

*$\boxed{2^{ND}}$ Trace 5 enter, enter, enter

Graphical Interpretation of Solutions

For a system of two linear equations in two variables, the number of solutions is one of the following.

Number of Solutions	*Graphical Interpretation*
1. Exactly one solution	The two lines intersect at one point.
2. Infinitely many solutions	The two lines are coincident (identical).
3. No solution	The two lines are parallel.

Example 7 (an application)

w = wind speed

P = plane speed

An airplane flying into a headwind travels the 2000-mile flying distance between Cleveland, Ohio and Fresno, California in 4 hours and 24 minutes. On the return flight, the same distance is traveled in 4 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

$\frac{24}{60}$

Time × Speed = distance

$\frac{4.4(p-w)}{4.4}=\frac{2{,}000}{4.4}$

$p-w=454.5$

$\frac{4(p+w)}{4}=\frac{2{,}000}{4}$

$p+w=500$

$\frac{2p}{2}=\frac{954.5}{2}$

Assignment: Page 491, #7-15odd, 25, 27, 41-45odd, 55, 61, 71, 74 Thank you ☺

$p=477.25$ mph, $w=22.75$ mph