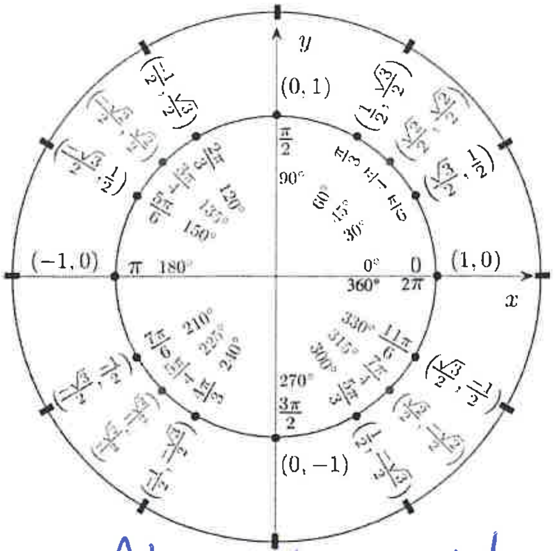
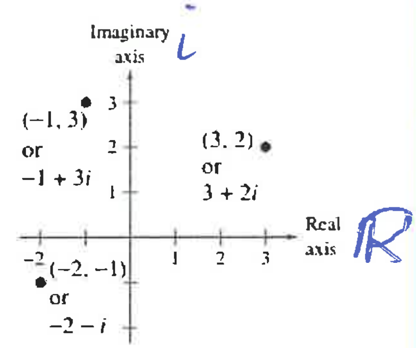


Name 2/11/2020

HansenMath™ Pre-calc: 6.5 Trig form of Complex Numbers

Recall that you can represent a complex number z = a + bi as the point (a, b) in a coordinate plane (the complex plane). The horizontal axis is called the *real axis* and the vertical axis is called the *imaginary axis*.

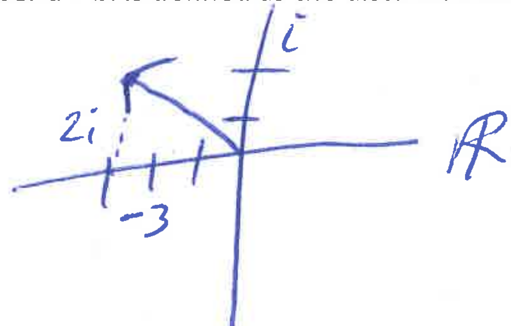


The Absolute Value of the complex number a + bi is defined as the distance between the origin (0, 0) and the point (a , b).

Thus, if follows that |a + bi| = $\sqrt{a^2+b^2}$

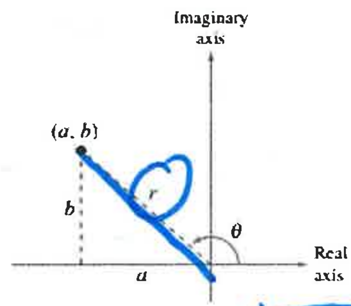
Example 1: Plot z = -3 + 2i and find its $\boxed{\text{absolute value.}}$

$$|z| = \sqrt{(-3)^2+(2)^2} \rightarrow \sqrt{13}$$



Just as we saw with normal vectors, it can be helpful to write complex numbers in trigonometric form.

> **Trigonometric Form of a Complex Number**
>
> **The trigonometric form** of the complex number $z = a + bi$ is given by
>
> $z = r(\cos\theta + i\sin\theta)$
>
> where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2+b^2}$, and $\tan\theta = b/a$.
> The number r is the **modulus** of z, and θ is called an **argument** of z.



Goal

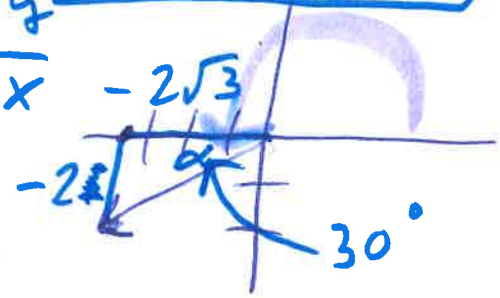
Example 2: Write the complex number $z = -2\sqrt{3} - 2i$ in trigonometric form. → $\boxed{r(\cos\theta + i\sin\theta)}$

$r = \sqrt{(-2\sqrt{3})^2+(-2)^2}$ $\quad \tan\alpha = \frac{\cancel{-2}}{\cancel{-2}\sqrt{3}} \rightarrow \frac{1}{\sqrt{3}} \quad \frac{y}{x}$

$r = \sqrt{12+4}$ $\quad \theta = 180 + 30^\circ = 210^\circ$

$r = \sqrt{16}$

$r = 4$ $\quad$ * $4(\cos 210^\circ + i\sin 210^\circ)$



Example 3: Write the complex number $z = \sqrt{6}(cos(-\frac{\pi}{4}) + isin(-\frac{\pi}{4}))$ in standard form a + bi

No cos, sin

$$\sqrt{6}\left(\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}i\right)$$

$$\frac{\sqrt{12}}{2} + -\frac{\sqrt{12}}{2}i \rightarrow \frac{\cancel{2}\sqrt{3}}{\cancel{2}} - \frac{\cancel{2}\sqrt{3}}{\cancel{2}}i \rightarrow \sqrt{3} - \sqrt{3}i$$

Trigonometric form adapts nicely to multiplication and division of complex numbers:

Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be complex numbers.

$$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)] \quad \textbf{Product}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)], \quad z_2 \neq 0 \quad \textbf{Quotient}$$

Let's try it: $z_1 = -3(cos(\frac{4\pi}{3}) + isin(\frac{4\pi}{3}))$ $z_2 = 7(cos(\frac{\pi}{2}) + isin(\frac{\pi}{2}))$

Example 4: Find z_1z_2

$$(-3)(7)\left[\cos\left(\overset{8}{\frac{4\pi}{3}}+\overset{3}{\frac{\pi}{2}}\right) + i\sin\left(\frac{4\pi}{3}+\frac{\pi}{2}\right)\right]$$

$$-21\left[\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right]$$

$$-21\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$$

$$-\frac{21\sqrt{3}}{2} + \frac{21}{2}i$$

Example 5: Find $\frac{z_1}{z_2}$

$$-\frac{3}{7}\left[\cos\left(\frac{4\pi}{3}-\frac{\pi}{2}\right) + i\sin\left(\frac{4\pi}{3}-\frac{\pi}{2}\right)\right]$$

$$-\frac{3}{7}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right]$$

$$-\frac{3}{7}\left[-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right]$$

$$\frac{3\sqrt{3}}{14} - \frac{3}{14}i$$

Assignment: Page 456, #7, 8, 13, 16, 17, 31, 33, 37, 43, 45 ☺