Name: 2/7/20

HansenMath™ Pre-calc: 6.4 Vectors and Dot Products, part 1

In this chapter on vectors, we have looked at addition of vectors and scalar multiplication of vectors.

Both of those aforementioned operations yield another vector.

In this section, we will study yet another vector operation, called the Dot product.

The dot product yields a scalar, rather than a <vector>.

The dot product can be thought of as the magnitude of one vector multiplied by the magnitude component of another vector in the same direction, or $\|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta$. Let's take a look at two examples.

Example 1

$\mathbf{u} = \langle 3, 4 \rangle$ $\mathbf{v} = \langle 5, 0 \rangle$

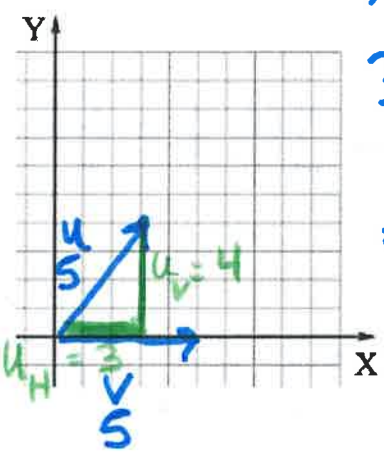
DoT product

$x_1 \cdot x_2 + y_1 \cdot y_2$

$3 \cdot 5 + 4 \cdot 0$

$15 + 0$

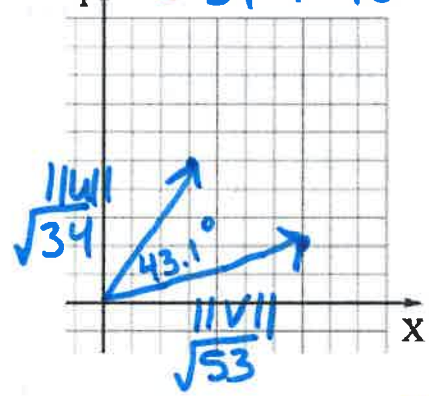
$= 15$ DoT product



Example 2

$\mathbf{u} = \langle 3, 5 \rangle$ $\mathbf{v} = \langle 7, 2 \rangle$

$u \cdot v = 3 \cdot 7 + 5 \cdot 2$
$= 21 + 10$ $= \boxed{31}$



oR $\|u\| \cdot \|v\| \cdot \cos\theta$

$\sqrt{34} \cdot \sqrt{53} \cdot \cos 43.1^\circ$

$= 31$ (same as $u \cdot v$)

Definition of Dot Product

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

Properties of the Dot Product (See the proofs on page 471.)

Let **u**, **v**, and **w** be vectors in the plane or in space and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{0} \cdot \mathbf{v} = 0$
3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$

DoT product

Example 3: Find $(u \bullet v)v$ if $\mathbf{u} = \langle 2, 1 \rangle$ and $\mathbf{v} = \langle 3, -4 \rangle$

$2 \cdot 3 + 1 \cdot -4$

$6 + -4$

$(u \cdot v) = 2$

$2(v)$

$2\langle 3, -4 \rangle = \langle 6, -8 \rangle$

* optional

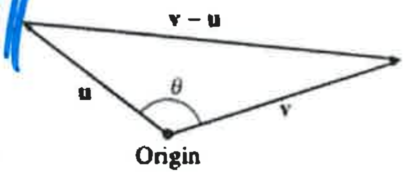
The dot product of a vector **u** times itself is $||\mathbf{u}||^2$ Sounds strange? Let's look at an example:

Example 4: Use the dot product to find $||\mathbf{u}||$ when $\boxed{\mathbf{u} = \langle 2, -4 \rangle}$ $\sqrt{20}$

$$u \cdot u = ||u||^2$$

$$4 + 16 = 20 = ||u||^2 \rightarrow \sqrt{||u||^2} = \sqrt{20} = ||u||$$

The dot product is very useful to find the angle between two (non-zero) vectors.



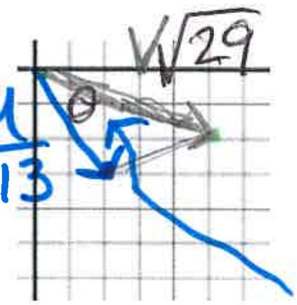
* **Angle Between Two Vectors** (See the proof on page 471.)

If θ is the angle between two nonzero vectors u and v, then

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}$$

← Dot product
← magnitudes

* use to Find θ



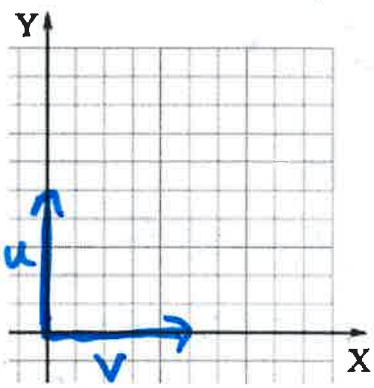
Example 5: Find the angle between **u** = 2i - 3j and **v** = 5i - 2j

$u = \langle 2, -3 \rangle$ $v = \langle 5, -2 \rangle$

$$\cos\theta = \frac{2\cdot 5 + -3\cdot -2}{\sqrt{13}\cdot\sqrt{29}} \rightarrow \frac{16}{19.416} = \cos\theta$$

$$\cos^{-1}\left(\frac{16}{19.416}\right) = 34.5^\circ = \theta$$

Example 6: Given **u** = < 0, 5 > and **v** = < 5, 0 > find u • v and also the angle between **u** and **v**.



90° angle θ

$u \cdot v$

$0\cdot 5 + 5\cdot 0 = 0$

Test: if Dot prod. is ∅ you get a 90° Angle!

Notice that these two above vectors are perpendicular to each other.

Another word for this is Orthogonal, meaning vectors which meet at 90°.

Example 6: Are the vectors **u** = < 2, -3 > and **v** = < 6, 4 > orthogonal? Show evidence.

$$u \cdot v = 2\cdot 6 + -3\cdot 4$$

$$12 + -12 = \emptyset$$

yes 90° orthogonal

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