Name: 2/6/2020

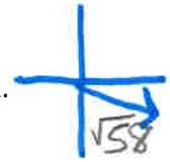
HansenMath™ Pre-calc: 6.3 Vectors in the Plane Notes, Part 2

In many applications of vectors, it is useful to find something called a "unit vector" that has the same direction as a given nonzero vector **v**.

A **unit vector** is a vector with the same direction as the original vector, with a magnitude of 1.

u = unit vector = $\frac{V}{\|V\|}$ = $\boxed{\frac{1}{\|V\|} \cdot V}$ * To find "unit vector"

Example 4: Find a unit vector **u** in the direction of vector **v** = <7, -3>. Then verify it has a magnitude of 1.



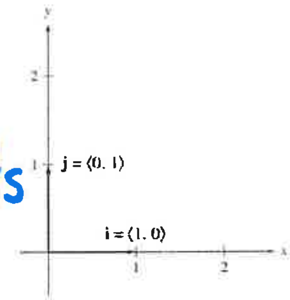
$\|V\| = \sqrt{(7)^2 + (-3)^2} = \sqrt{58}$ magnitude of V

$\frac{1}{\sqrt{58}} \cdot \langle 7, -3 \rangle = \langle \frac{7}{\sqrt{58}}, \frac{-3}{\sqrt{58}} \rangle$ unit vector ← prove Length of 1

$\sqrt{\left(\frac{7}{\sqrt{58}}\right)^2 + \left(\frac{-3}{\sqrt{58}}\right)^2} = \sqrt{\frac{49}{58} + \frac{9}{58}} = \sqrt{\frac{58}{58}} \rightarrow 1$

The unit vectors <1, 0> and <0, 1> are called the standard unit vectors

i = < 1 , 0 > and **j** = < 0, 1 >



These vectors can be used to represent any vector **v** = <v_1, v_2> as: v_1**i** + v_2**j**

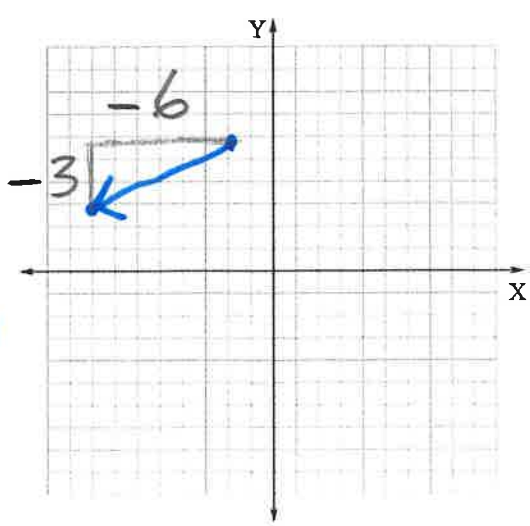
The scalars v_1 and v_2 are called the horiz. & vert. components

The vector sum v_1**i** + v_2**j** is called a linear combination of vectors **i** and **j**.

Example 5:

Let **u** be a vector with ***initial point*** (-2, 6) and ***terminal point*** (-8, 3).

Write **u** as a linear combination of the standard unit vectors **i** and **j**.



Component Form: <-6, -3>

linear combination: -6i + -3j

OR -6i - 3j

Example 6: Let **u** = **i** + **j** and **v** = **5i** - 3**j**. Find 2**u** - 3**v**.

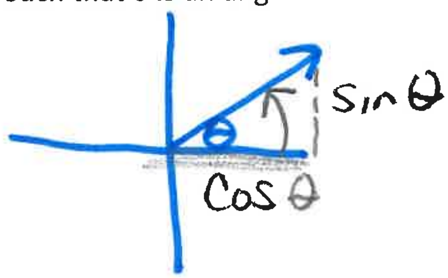
$u = \langle 1, 1 \rangle \qquad v = \langle 5, -3 \rangle$

$2u = \langle 2, 2 \rangle + 3v = \langle -15, +9 \rangle$

$= \langle -13, 11 \rangle$ or $-13i + 11j$

Direction Angle:

If **u** is a unit vector, such that θ is an angle that measures counter-clockwise from the positive x-axis to **u**:



θ is direction angle of vector **u**.

u = $\langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle = \cos\theta\, i + \sin\theta\, j$

Given **u** is a vector with direction angle θ. If **v** = a**i** + b**j** has direction angle θ

then **v** = $\|v\| \langle \cos\theta, \sin\theta \rangle = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$

To find the direction of **v**: θ

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} = \frac{\|v\|\sin\theta}{\|v\|\cos\theta}$$

Example 7: Find the direction angle of **v** = -6i + 6j

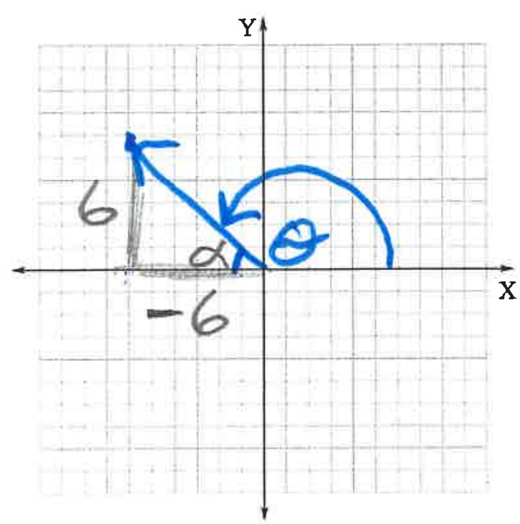
Component: $\langle -6, 6 \rangle$

$\tan\alpha = \frac{6}{-6}$

$\tan\alpha = -1$

$\alpha = \tan^{-1}(-1) = -45^\circ$

$180 - 45^\circ = 135^\circ = \theta$



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