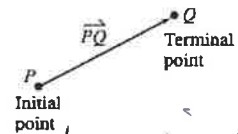
Name: 2/5/20

HansenMath™ Pre-calc: 6.3 Vectors in the Plane Notes, Part 1

Many quantities in geometry and physics, such as area, time, and temperature, can be represented by a single real number. Other quantities, such as force and velocity, involve both ***magnitude*** **and** ***direction*** and cannot be completely characterized by a single real number.



To represent such a quantity, you can use a directed line segment.

The directed line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q

Its Magnitude, or length (size), is denoted by $\|\overrightarrow{PQ}\|$ and can be found by using the Distance Formula (Or Pythagorean Theorem).

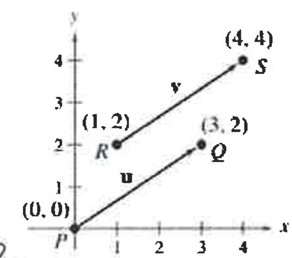
Two directed line segments that have the same magnitude and direction are Equivalent.

The set of all directed line segments that are equivalent to a given directed line segment, $\overrightarrow{PQ}$, is a vector **v** in the plane, written $\mathbf{v} = \overrightarrow{PQ}$.

Vectors are denoted by lowercase, bold letters such as **u**, **v**, and **w**.

Example 1 Equivalent Directed Line Segments

Let **u** be represented by the directed line segment from $P = (0, 0)$ to $Q = (3, 2)$, and let **v** be represented by the directed line segment from $R = (1, 2)$ to $S = (4, 4)$, as shown in Figure 6.17. Show that $\mathbf{u} = \mathbf{v}$.



Show they have same magnitude (length) & Same Direction (slope)

$d_v = \sqrt{(4-1)^2 + (4-2)^2} = \sqrt{13}$ $m_v = \frac{\Delta Y}{\Delta X} = \frac{4-2}{4-1} = \frac{2}{3}$

$d_u = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{13}$ $m_u = \frac{2-0}{3-0} = \frac{2}{3}$ Same Direction!

Component Form of a Vector

Standard position: Directed Line Segment with initial point at the Origin. If initial point is at (0,0) then coordinate of the terminal point is (v_1, v_2).

This is the Component Form of a vector, written as: $\mathbf{v} = \langle v_1, v_2 \rangle$

Component Form of a Vector

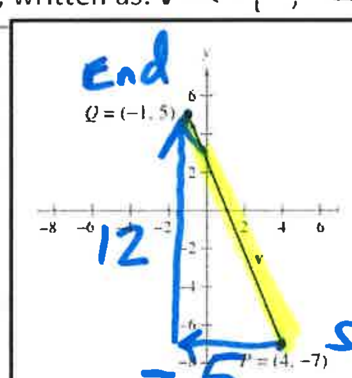
The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is given by

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$

The **magnitude** (or length) of **v** is given by

$$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

If $\|\mathbf{v}\| = 1$, **v** is a **unit vector.** Moreover, $\|\mathbf{v}\| = 0$ if and only if **v** is the zero vector **0**.



$\|\mathbf{v}\| = 13$

$\sqrt{169}$

Example 2: Find the component form and magnitude of the vector **v** that has initial points shown above.

$\mathbf{v} = \langle -1-4, 5 - -7 \rangle$ $\mathbf{v} = \langle -5, 12 \rangle$ $\|\mathbf{v}\| = \sqrt{(-5)^2 + (12)^2}$

Vector Operations:

Addition: To add vectors put <u>initial pt</u> of 2nd vector on the <u>terminal pt</u> of the 1st vector. (tip-to-tail method).

If **u** = < u_1 , u_2 > and **v** = < v_1 , v_2 > then **u + v** = < u_1+v_1 , u_2+v_2 >

Scalar Multiplication: Changes vector by length | k |. If k is <u>positive</u> then k**u** is in the same direction as **u**. But if k is <u>negative</u> then k**u** is in the <u>opposite</u> direction.

k times **u** = < ku_1 , ku_2 > = k**u**.

Subtraction: The difference of **u** and **v** is: **u - v = u +** ($-v$) = < $u_1 - v_1$, $u_2 - v_2$ >

Geometrically, use directed line segments with the <u>same</u> initial point. The difference **u - v** is the vector from the *terminal point* of **v** to the *terminal point* of **u**.

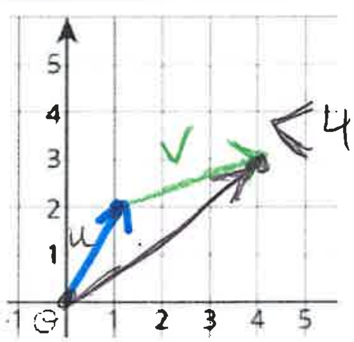
*** ***My suggestion for subtraction is to just stick to addition method.*** Ex.) Instead of **u - v**, draw as **u + (-v)**

Example 3: Let **u** = <1, 2> and **v** = <3, 1>. Find:

u+(−v) <−3,−1>

a.) **u + v**

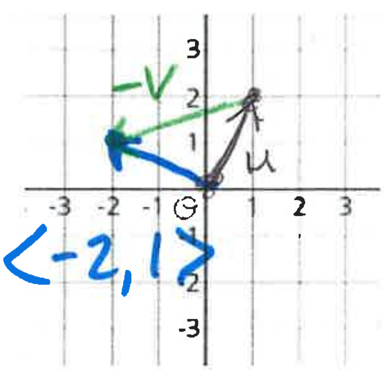
origin
<1,2>



<4,3>

b.) **u - v**

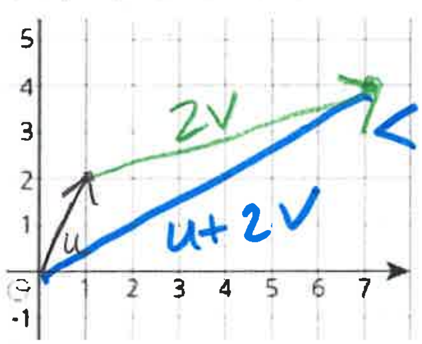
<1,2>



<−2,1>

c.) **u + 2v**

<6,2>

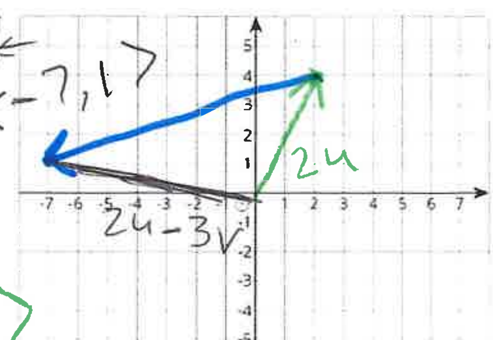


<7,4>

d.) **2u - 3v**

<−7,1>

−3v
<−9,−3>



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